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THE MATHEMATICS TEACHER

Volume XLV Number 2

The Place of Mathematics in Human Affairs, and Related Curriculum Problems*

By William Betz
Rochester, New York

Preliminary Considerations

One does not have to be a professional student of mathematics to become convinced that the bearings of mathematics have world-embracing dimensions. In an almost unique sense, this huge area of learning touches the entire physical universe, and it permeates many aspects of the industrial, technical, and social fabric which human ingenuity has created on this revolving globe of ours. Obviously, then, we must greatly restrict the scope of our discussion, confining ourselves to the more modest question, how the school can or should react to this dramatic story in the education of American youth.

Historians of mathematics have established a close relation of mathematics to human affairs, from the very dawn of human history. But they have also pointed out the significant fact that whenever mathematics became merely a tool for the solution of everyday problems, as in the days of ancient Egypt and of Rome, complete stagnation was the invariable result. It was only when the speculative minds of the Greeks began to look beneath the surface and to examine basic theoretical questions that genuine mathematical structures became possible. The tension between theory and practice has continued to this day, but with an important difference. For the men who work at the frontiers of human knowledge, there is no such conflict. They know only too well that both in science and in mathematics the contributions made by pure research, by "fundamental" investigations, when fruitfully utilized in laboratories and technological centers, have made possible the fairy-like transformations that we think of in connection with this power age, or this air age, or this atomic age. That is, behind the enormous scene of modern progress stand the trained minds of mathematical or scientific thinkers.

It would be a most rewarding and revealing task to examine in some detail the negative treatment accorded to mathematics during the severely practical ages, and to note, by way of contrast, the amazing periods of growth which always resulted when keen intellects succeeded in building an enlarged mathematical framework. Particularly fascinating would be the story of the past three centuries, beginning with the "century of genius," as Whitehead so aptly called the seventeenth century. It laid the foundation for our scientific and technological achievements and thus ushered in the modern era. The

* Based on an address at Eastern Illinois State College, Charleston, Illinois, June 20, 1951.

eighteenth century produced a galaxy of great mathematicians. Utilizing the mathematical instruments created by Descartes, Newton, and Leibniz, they were lured by the possibility of attempting a mathematical description of the universe. Typical of the period was the monumental French *Encyclopédie* of Diderot and others (35 volumes, 1751–1780), with its famous preface by D'Alembert. And among the culminating achievements of that century were Lagrange's *Mécanique Analytique* (1788) and Laplace's *Mécanique Céleste* (five volumes, 1799–1825). Then came the revolutionary nineteenth century which vastly extended the domain of pure and applied mathematics.

Throughout this phenomenal epoch, it was the mathematical theorists who prepared the way for the ever-increasing scientific, industrial, and economic progress achieved in the western world. Thus theory and practice were constantly interrelated, each fructifying and stimulating the other.

And today we are even more certain than was Gauss, over a century ago, of the predominant place of mathematics in the scheme of human knowledge. He referred to mathematics as the "queen of the sciences." For, as Professor Eric T. Bell has so forcefully stated it, on the basis of impressive documentary evidence, ours is the "Golden Age of Mathematics."[1] It would be next to impossible to exaggerate the impact of mathematics on our civilization.

Now, what are the lessons that modern education should derive from this factual record? Surely, no one who is familiar with the undeviating trends of this development can deny that *a citizen of the modern world cannot afford to be ignorant of mathematics, because the world in which we live is so largely mathematical.*

[1] Eric T. Bell, *Mathematics, Queen and Servant of Science* (New York: McGraw-Hill Book Co., 1951), pp. 6 ff. See also, Hermann Weyl, "A Half-Century of Mathematics," *The American Mathematical Monthly*, October, 1951.

I. Modes of Appraising the Role of Mathematics in the Modern World

Many and varied have been the approaches suggested by curriculum theorists for ascertaining or determining for each of our principal courses of study a convincing raison d'être. Thus, they speak of the "social values" approach, the "historical" approach, the "shortages" approach, the "job-analysis" approach, the "creative values" approach, and so on. In the field of mathematics, likewise, there have been earnest attempts to discover objective ways of appraising the place of that subject in human affairs, and to utilize the findings thus obtained in the construction of mathematical curricula. Among these mathematical approaches the following seem particularly significant.

1. The Elimination Idea. Here one asks the question, what would happen if all knowledge of mathematics were suddenly destroyed. The consequences of such an imaginary calamity have been described with characteristic clarity and penetration by Professor D. E. Smith. He explained what would be the result if tonight there should be wiped off the face of the earth every book on mathematics, every mathematical symbol of any kind, and every machine for computing or for recording numbers. The whole world would slow down, and trade would relapse into the condition of barter as in the days of savagery. Banking, building, engineering, and similar enterprises would awaken to a living death.[2]

Such a picture, fantastic and yet entirely truthful, will at least serve to outline in a broad way the all-pervading importance of mathematics in human affairs.

2. Studying Typical Areas of Living. Again, one may wish to find out what mathematical ideas, skills, kinds of under-

[2] David Eugene Smith, *The Poetry of Mathematics and Other Essays*, The Scripta Mathematica Library, Number One, New York, N. Y., pp. 16–17.

standing and appreciation, are a concern of the home, of business, of the principal professions, of citizenship, of leisure, and so on. To obtain this information, one would have to interview many groups of people, such as homemakers, businessmen, builders, contractors, bankers, economists, scientists, statisticians, accountants, pilots, and so on. The danger of this approach, as we shall see, is that of giving undue weight to considerations of mere frequency of occurrence or of use.

3. Looking at Vocational Needs. This is really a corollary of the preceding approach. Thus, a detailed analysis of every mathematical item entering into the daily work of a mechanic, a carpenter, a machinist, a draftsman, and so on, was considered desirable a generation ago, as a basis for the formulation of "personalized" mathematical curricula. But this program was eventually seen to be a hopelessly complex one. The school cannot prepare its body of students for an endless array of specific vocational skills. Instead, we are interested today in the "common learnings" found necessary in a large variety of life situations. There can be no doubt whatever that an appreciable fraction of these essential common learnings involves at least elementary literacy in mathematics. Curiously enough, this fact is often overlooked.

4. Depending on Authoritative Check Lists. From such analyses there have emerged a variety of check lists. They differ from those of the past in that they have been ascertained by various investigators on the basis of more or less exhaustive studies of contemporary life. One such list is that issued by the Commission on Post-War Plans of The National Council of Teachers of Mathematics.[3] Teachers, curriculum committees, and authors will derive much benefit from a careful study of such lists.

5. A Comprehensive Worldwide Analysis of Mathematical Backgrounds. Anyone who has attended recent meetings of The National Council of Teachers of Mathematics has no doubt become aware of the widespread attempts to publicize and dramatize the role of mathematics in modern life. Among the promising tools of this publicity campaign are the yearbooks of the National Council, as well as numerous monographs, films, school plays, and the like.

In view of all that has been said thus far it is rather amazing that a group of very vocal educational critics persist in their negative attitude toward mathematics. They keep repeating their usual thesis that beyond some basic work in arithmetic and its common socialized applications all further training in mathematics is of consequence only for "the few." Moreover, they have no respect for the evidence which points in the opposite direction. They characterize as purely "subjective," and hence as unreliable, the analyses which are favorable to mathematics. They question even the most authoritative check lists. And, of course, they reject or openly ridicule the alleged *cultural* values of mathematics. How can one deal with such a situation?

Clearly, the case must be appealed to a higher tribunal of competent judges. Such an appeal has occurred. At the 1950 International Congress of Mathematicians at Harvard University, active steps were taken to bring about a worldwide appraisal of the status of mathematics in all the participating nations, with particular reference to the actual importance of mathematics in human affairs. Resolutions pertaining to this matter were adopted at the second session of Section VII of the Congress. These resolutions called on the new International Mathematical Union to initiate this program through the continuation and revitalization of the International Commission on the Teaching of Mathematics.[4]

[3] See the November, 1947 issue of THE MATHEMATICS TEACHER.

[4] William Betz, "Mathematics for the Million, or For the Few?" THE MATHEMATICS TEACHER, XLIV (January, 1951), 21.

It is to be hoped that when these resolutions are actually carried out, the findings thus obtained will serve as a basis for more informed and hence more authoritative mathematical curricula throughout the world.

II. The Need of Adequate Criteria of Selection

Studies such as those suggested above should prove to the most obdurate skeptic that today the scope of mathematics is coextensive with that of human culture. But it should be equally obvious that the school cannot hope to cope with such a vast setting, any more than it can cover the whole domain of science. At best, it can offer only a modest introduction to the stupendous realm of modern mathematics and to the encyclopedic range of its applications. That is, the mathematical program of the school involves the judicious use of certain criteria of selection, of which the following are of special importance.

1. Frequency of Use. This is certainly a most incisive arbiter in deciding whether a particular mathematical item should be included in the mathematical program of the school. For years, curriculum theorists have been disposed to give as much weight to this criterion, in the field of mathematics, as they have attached to a frequency index in Thorndike's famous word lists. And there is no desire to minimize the value of frequency studies. They helped to eliminate from our curricula much traditional material that really served no useful purpose. On the other hand, a crass and purely utilitarian emphasis on immediate and wide applicability has done an enormous amount of harm. It has inflicted grave and unwarranted injury on virtually all cultural aspects of education. It has led to a severe curtailment of instruction in such fields as history, literature, and foreign languages, to the detriment of our national culture. In mathematics, the resulting attitude of uninformed extremists is reflected in the following typical assertions of a well-known educator, as published in his Inglis Lecture at Harvard University.

"No youth under eighteen, male or female, ever uses or encounters, outside the classroom, the theorems of either plane or solid geometry. For that matter, not one man in a thousand and probably not one woman in a hundred thousand ever finds them useful in either the general or wage-earning affairs of life. The truth is that Euclidean geometry is not a genuine subject, valuable to everybody, but is a specific vocational study of real value only to prospective teachers of the subject. . . .

"Where can any high school student find a place where the theorems of geometry are being used outside the school room? What meaning and use do they have in his life? . . .

"All that has been said regarding geometry applies with about equal force to algebra, which is almost as fully divorced from adolescent life."

All this is prefaced by a vigorous attack on the entire concept of the transfer of training. The same educator then applies to the traditional high school subjects the yardstick of "comparative use value." He is thus led to the conclusion that "on the basis of this rating, all foreign languages and all mathematics should be dropped from the list of required college-preparatory studies."[5] In vocational courses, he feels, the small amount of necessary mathematics can readily be picked up in a few hours. Preachments of this sort were warmly approved by the leading educational policymakers. These declarations are directly responsible for the fact that today very large numbers of our secondary students are no longer exposed even to a single year of mathematics. The most recent pronouncements of the Commission on Life Adjustment for Youth, sponsored by the United States Office of Education, still reflect essentially the same position.

[5] Charles Allen Prosser, ***Secondary Education and Life*** (Harvard University Press, 1939).

2. Significance. It is obvious, then, that mere frequency of occurrence or of use is not a dependable criterion of selection in the construction of a curriculum. The factor of *significance* or *indispensableness* must be added. Not merely what *is*, but what *ought* to be, must be given due weight. That fact has often been forgotten. It should be stressed again and again. In the early days of the curriculum revision movement, none other than Professor Harold Rugg wrote this pertinent word of caution which is needed today more than ever:

"The day has passed in which a single individual professor, teacher or administrator, psychologist, educational lawgiver or research specialist, can hope to master the manifold, highly professional tasks of curriculum-making. They are far too difficult and complex for any one person to hope to compass them all single-handed. . . .

"*The task of stating the goals of education is not to be consummated by an analysis of social activities alone. It will be aided by the latter, but must not be dominated by it. It will be achieved only by hard thinking and by the most prolonged consideration of facts by the deepest seers of human life.* For the great bulk of our curriculum, therefore, the analysis of social activities will influence the judgments of frontier thinkers; *but it is the judgment of the seer based upon the scientific study of society—not the mere factual results of social analysis—that will determine the more intangible, but directing materials of our curriculum.*

"*Social analysis merely gives us the techniques and knowledges we should have on tap. For the basic insights and attitudes we must rely, as we do for the statements of the goals of education, upon human judgment. It is imperative, however, that we make use of only the most valid judgments. The forecasting of trends of social movement, underlying them, demand erudition and maturity of reflection that eventuates only from prolonged and scientific study of society. To the frontier of creative thought and of deepest feeling we go for guidance as to what to teach.*"[6]

3. Simplicity. Because of our congested time schedules, and of the inadequate preparation of many pupils, constant attention must also be given to the weighty question of *simplicity* and of *economy of time.* This criterion rules out not merely complicated types of technique, but also many significant and interesting applications because their background is too involved to make possible even a fair degree of understanding and mastery in the average classroom. It is most gratifying, of course, that a profuse literature now exists on ways and means of vitalizing the teaching of mathematics.[7] What has *not* been accomplished, however, is a corresponding absolutely necessary extension of the time required to incorporate more of these promising modes of enrichment in our crowded curricula.

Other criteria of selection might readily be suggested. Thus, a decade ago one basic curriculum study listed over one hundred criteria considered necessary in the evaluation of "teaching and learning materials and practices."[8] Today we are satisfied with a much smaller list. For our purposes the three criteria discussed above are sufficiently adequate.

III. Which Fields of Application Meet These Criteria?

The criteria we have mentioned apply to all the materials of instruction incorporated in our curricula. However, we are now face to face with the critical problem of designating such mathematical *applications* as would seem to comply with these

[6] Harold Rugg, "The School Curriculum and the Drama of American Life." *The Twenty-Sixth Yearbook of the National Society for the Study of Education* (Bloomington, Ill.: Public School Publishing Company), pp. 52 ff.

[7] See, for example, the Seventeenth and Eighteenth Yearbooks of The National Council of Teachers of Mathematics.

[8] Herbert B. Bruner and others, *What Our Schools Are Teaching* (New York: Bureau of Publications, Columbia University, 1941), pp. 211–20.

criteria. It is not a difficult task for an informed student of mathematics to draw up an extensive list of potentially valuable applications. And the lists that have been published from time to time have not been without value. But until it is shown, on the basis of actual teaching schedules and of test results, how such applications can be incorporated into the daily routine of the school, and to what extent they are understood and mastered by the pupils, very little has been accomplished. Theorists have a fatal way of ignoring the chronic road-blocks that have prevented the achievement of functional competence in mathematics. For nearly six decades there has been a continuous outcry against the compartment system,—a year of algebra followed by a year of geometry—, but it is still with us. The four-year high school, long known to students of education as a "historic accident," with every passing year gets more out of step with the educational needs of our age. To these handicaps have been added the totally indefensible postponement of arithmetic and, in consequence, the wrecking of junior high school mathematics.

And so, our "radius of action" is still greatly limited. In the main, there are three principal fields, more than any others that might be suggested, which come within the scope of the criteria pointed out above. These fields are represented by the elementary aspects of finance, of technology, and of science. Each of these domains plays a most important role in human affairs. Let us briefly examine the contributions that each can and should make to the mathematical program of the school.

1. Financial Applications. This type of work is certainly of enormous extent. It ranges all the way from ordinary "grocery store arithmetic" to the most involved problems of economics and of "actuarial mathematics." In the school we must give due attention to considerations of wise buying, including installment buying. Again, a careful study of personal and household budgets is of central importance. Moreover, having a savings account and providing for personal and financial protection through such means as insurance should not be neglected. The mathematics of community life, with its constant demand for adequate funds to be supplied by taxation or voluntary contributions, is a concern of every citizen. This is also true of the financial transactions having to do with home ownership and with carefully considered types of investments.

No agreement has been reached as to the precise grade placement of these financial applications. However, the idea of postponing their study to the last year of the high school seems decidedly unwise. Very many of the students who need this kind of information and training have left the school by that time or insist on sidestepping the opportunity of obtaining this training, at such a late date. Hence it is a far better plan to include some of the essential phases of "financial mathematics" continuously, each year making its proper contribution to a growing insight and mastery.

2. Technical or Vocational Applications. While the emphasis on "money problems" has often been carried to the point of creating the impression that the school is primarily concerned with "dollar mathematics," this is far from being the case in the field of even the lower phases of technical or shop mathematics. Here, much reorientation is necessary. In a recent year American industries employed 26% of the total labor force, more than 16 million persons, and more than 12 million workers earned their living in some trade. Should not all our mathematical courses, whether of the first track or the second track, take account of these impressive figures?

It will take time and much experimentation to bridge this serious gap in our curricula. But even now, it is fairly certain that for the purposes of general education the following items will have to be included in practically all of our courses, beginning in the seventh grade: (1) direct

and indirect measurement, with due attention to questions of accuracy, precision, and tolerances, together with at least an introduction to approximate computation; (2) the ready use of standard formulas of mensuration; (3) the techniques of geometric construction, supplemented by some elementary work in mechanical drawing; (4) a familiarity with working drawings and blueprints; (5) the ability to solve elementary problems having to do with such mechanical devices as the lever, the inclined plane, and pulleys and gears.

Specialized shop courses should, of course, offer a more extended program. But until we obtain more liberal time allotments, the brief list suggested above may be regarded as a minimum vocational equipment of the average student.

3. Science Applications. Mathematics has been called the "hand-maiden of science." The phenomenal development of science, since the days of Galileo, would have been impossible without the helping hand of mathematics. The huge structure of modern science rests on the labors of mathematical giants, men like Newton, Euler, Laplace, Gauss, and Maxwell.

The close connection of mathematics and science has caused, in some of the leading nations, a definite correlation of these two fields in the curricula of the schools. Determined efforts have long been made to bring about a similar affiliation in our American schools,—thus far without success. The chief credit for keeping alive a demand for this essential reform belongs to the Central Association of Science and Mathematics Teachers. From its Fiftieth Anniversary Volume we quote the following revealing statements:

"By the turn of the century, science had set its roots deep in the society of America. Scientific discoveries became topics of everyday conversation. The experimental method of science rapidly gained popular approval and acceptance. A golden age of science—a phenomenon of western civilization—was at hand.

"This rapid growth and public acceptance of science, coupled with a rapid increase in the enrollment of secondary schools, stimulated new development in the teaching of mathematics and the sciences.

"On the crest of this wave of scientific interest and emphasis, a group of physics teachers from schools in the Central States met in Chicago in the spring of 1902 to consider the organization of an association of physics teachers. . . .

"The fact that the scientific impetus which stimulated the organization of physics teachers influenced other areas is evidenced in the action taken by the Mathematics Section of the Educational Conference of Academies and High Schools in November of the same year. These teachers were concerned with the improvement of instruction in mathematics by introducing the laboratory method and by bringing about a closer correlation of mathematics with other subject matter of the curriculum, especially physics.

"To this end, . . . plans were formulated to include mathematics and the other science fields in the April meeting of the Physics Association. This meeting, held at the Armour Institute of Technology in Chicago, April 9–11, 1903, was the culmination of the unification movement. The larger organization was named the Central Association of Science and Mathematics Teachers."[9]

Enthusiastic supporters of the correlation movement went so far as to advocate the virtual fusion of science and mathematics. A syllabus tending in that direction was actually prepared and published.[10] It would be of more than historic

[9] See *A Half Century of Science and Mathematics Teaching* (Oak Park, Illinois: Central Association of Science and Mathematics Teachers, Inc., 1950), pp. 1–2.

[10] This document was issued as an "Appendix of the Proceedings of the Second Annual Meeting of the Central Association of Science and Mathematics Teachers." Second Edition, 1905. It had the title: *Report of the Committee on the Correlation of Mathematics and Physics in Secondary Education.*

interest if a reprint of that document were made available at this time.

The movement failed because the fatal road-blocks, mentioned above, could not be eliminated. As a matter of fact, while there has been a growing emphasis on a largely descriptive type of general science, the enrollment figures in physics have been declining.

The present state of affairs cannot last indefinitely. The initiative in correcting the neglect of science must come from individual schools and teachers. It is entirely feasible even now, with an average group of pupils, to attempt a few simple laboratory experiments concerned with an explanation of the scientific method. The mathematical tools used by the scientist should be pointed out. Measurements involving metric units should be stressed. Problems based on such standard backgrounds as Hooke's law, Ohm's law, the laws of motion, the use of vectors, and the parallelogram of forces, can readily be used to replace some of the traditional, non-functional problem material.

There is every indication that a reasonable emphasis on the program of applications suggested in these pages is becoming standard practice in forward-looking schools. More than that, ambitious teachers have gone beyond these minimum goals, by adding units in aviation and in statistics, usually with gratifying success. And as soon as we shall have achieved continuous curricula, the door will be wide open for larger developments.

Conclusion

It remains to say a further word about the task of the school in giving to our American youth a more adequate conception and a more functional realization of the actual role of mathematics in modern life. The "muscular" mathematics which is still in vogue in a large number of classrooms ignores the immense cultural significance of mathematics. Mere manipulation is educationally barren and defeats the real objectives of mathematical instruction.

A decided reorientation is necessary if we would teach mathematics into its rightful place. The reports of national committees, as well as a long series of helpful suggestions in the professional literature, can aid powerfully in creating that new "mathematical atmosphere" which is of such vital importance. Visual aids, films, models, laboratory techniques, exhibits, and the like, are making valuable contributions in the desired direction. In the last analysis, however, all genuine reform can be effected, in each classroom, only by the informed mind, the sustained enthusiasm, and the creative skill of a real teacher.

HAVE YOU SEEN?

In *The Mathematics Student* for September–December, 1950:
"Patiganita and the Hindu Abacus" by R. Venkatachalam Iyer
"A Note on 'Four Fours'" by G. C. Patni.

In *American Journal of Physics* for December, 1951:
"An Early Experimental Determination of Snell's Law" by J. W. Shirley.
"On the Analysis of Transfer of Training" by L. A. Dexter and R. A. Thornton.
"Student Questionnaires as an Aid to Laboratory Teaching Techniques" by W. Geer.

In the Fall, 1951 issue of *The Pentagon:*
Bibliographies for topics for mathematics club programs: the magic number nine (7 references), plays (44 references), the story of pi (34 references).
"Angle Trisection" by Wanda Ponder.
"On Extremizing Polynomials without Calculus" by H. J. Hamilton.
"A Class of Internal Triangles" by H. T. R. Aude.
"Algebra Today and Yesterday" by Gertrude V. Pratt.

In *Pi Mu Epsilon Journal* for November, 1951:
"An Interesting Theorem" by P. A. Piza. (One hundred ninety-two times the cube of the sum of the first x squares is equal to the sum of the cubes of the first $2x$ triangular numbers plus twice the sum of their biquadrates.)

Minimum Mathematical Needs of Prospective Students in a College of Engineering

By KENNETH B. HENDERSON *and* KERN DICKMAN
College of Education, University of Illinois, Urbana, Illinois

THERE ARE several reasons why some students enter a college of engineering lacking adequate preparation in mathematics. One is that the mathematical needs of such students have not been clearly defined. It seems to be an auspicious hypothesis to assume that, if these needs are identified in some specificity and high school mathematics teachers apprized of them, students can be better prepared for collegiate work. Acting on this hypothesis, a study was conducted to discover the minimum mathematical needs of students who expect to enter the College of Engineering of the University of Illinois. Since the curricula and course content of most colleges of engineering tend to be similar, it is assumed that, in the absence of other data, these needs will serve very well to indicate "what it takes" in most colleges of engineering.

The validity of the results of a study like the present one, which utilized the judgmental and consensual processes, depends largely on the validity of the processes themselves. People, other than those who conducted the study, will evaluate the results in terms of how valid they consider the processes to be. To provide an opportunity for such an evaluation, the procedure of the study will be described.

The criterion of usefulness was assumed to be a major one (but not the only one) in the present study. Therefore, data were sought concerning the utility of various mathematical concepts, principles, and processes. Further, it was believed that as the basis for judgment is broadened the validity of the conclusions is enhanced. Accordingly, the final judgments were made by a committee composed of two members of the College of Engineering, two members of the Department of Mathematics, and two members of the College of Education interested in the teaching of mathematics.* It was felt that each of these groups of individuals had a point of view which was necessary in the judgmental process.

TWO SURVEYS AT THE UNIVERSITY OF ILLINOIS

At the outset the writing and research related to the problem were studied. These indicated that there are many topics in the high school mathematics curriculum which, according to a consensus of authorities, are indispensable for prospective engineering students. There are, however, other topics concerning whose importance there is a difference of opinion. It seemed advisable, therefore, to secure additional information. For this purpose two studies were set up.

The first of these studies consisted of a series of interviews with students enrolled in the College of Engineering. The purpose of these interviews was to determine the usefulness of selected mathematics topics in freshman and sophomore courses. Fifty persons were selected at random from the freshman class and fifty from the sophomore class. For purposes of comparison, twenty-five upperclassmen and thirty staff members also were interviewed.

From a comprehensive list of mathematical topics, a selection of topics was

* Professor Kenneth B. Henderson, Chairman of the Committee, College of Education; Professor William A. Ferguson, Department of Mathematics; Professor Randolph P. Hoelscher, Head of the Department of General Engineering Drawing, College of Engineering; Professor Bruce Meserve, Department of Mathematics; Professor Milton O. Schmidt, College of Engineering; Mr. Kern Dickman, Graduate Assistant, College of Education.

made for which information seemed desirable. The topics selected were phrased in the form of questions, such as, "Do you have to use third-order determinants?" In addition to answering the questions, the students were encouraged to comment about their own mathematical preparation. The answers to the selected questions as well as the comments of the students were recorded and later tabulated. The mean number of responses (indicating usefulness) in per cent was 39 for the freshmen, 56 for the sophomores, and 69 for the upperclassmen. It is evident that the amount of mathematics considered useful increases as the student progresses.

There was substantial agreement among the various groups with respect to the relative usefulness of the mathematical topics. The rank order of the topics was determined for each group of respondents, and Spearman's rank correlation coefficient was calculated. Although the topics cannot be assumed to be equally spaced with respect to usefulness, nevertheless, these rank correlation coefficients provide some measure of the agreement. Table 1 shows these calculated values.

TABLE 1

Groups	r
Instructors and Freshmen	.78
Instructors and Sophomores	.92
Freshmen and Sophomores	.81
Sophomores and Upperclassmen	.89

The second study consisted of a survey of the opinions of all staff members of the College of Engineering who were teaching freshman or sophomore courses, and of a selected group of full-time staff members of the Department of Mathematics. The purpose of the survey was to secure judgments concerning the importance for engineering students of various topics in the high school mathematics curriculum.

The survey was conducted by means of a questionnaire distributed by mail. The respondents were instructed to indicate by checking whether each topic is "indispensable" to engineering study, "desirable but not essential," or "unnecessary." Sixty per cent of the questionnaires were returned.

In every questionnaire study there is always the question of how carefully the respondents consider their answers. In an attempt to assess this in each of the two surveys, five topics were included which other studies indicated were indispensable. Five other topics which were agreed to be decidedly unnecessary were also included. It was assumed that a respondent, who marked "indispensable" those topics which were postulated as indispensable and "unnecessary" those postulated as unnecessary, carefully considered all the responses he made. It was gratifying to find that the great majority of respondents fell into this category. Hence, it was felt that the results were valid measures of the the judgments of the students and of the staff of the College of Engineering and the selected sample of the members of the Department of Mathematics.

WHAT HIGH SCHOOL MATHEMATICS TEACHERS HAD TO SAY

Using the findings of other investigators and those of the two studies conducted at the University of Illinois, the Committee selected topics and classified them into two categories: 1) those which were considered indispensable; and 2) those which were considered supplementary.

It was felt that the opinions of high school teachers of mathematics should be obtained. These are the people who help prepare the students for their engineering courses. They understand the psychology of the high school student and the realities of the public school situation better than most college professors do. Hence, it was assumed that they would be able to say whether what was tentatively proposed in the form of the list of topics could actually be accomplished in a four-year high school mathematics program.

Acting on this assumption, a group of nine representative high schools in Illinois

was selected. The mathematics teachers in these schools were either interviewed in a group or sent questionnaires. The teachers were asked to criticize the list of indispensable and supplementary topics. They were also asked whether in their opinion the set of topics labeled "indispensable" could be covered in a four-year high school mathematics program.

The teachers had minor suggestions to make, usually suggesting that certain topics be made more specific. It was the unanimous opinion of the teachers that the topics classified as "indispensable" could be covered in a four-year mathematics program. In fact, most of the nine high schools were presently covering many of the supplementary topics, and all but two or three of the topics labeled "indispensable." From the opinions of the teachers, it seemed reasonable to assume that the list was a realistic statement of what the high school could do to prepare prospective students of engineering.

Final Steps

Using the reactions obtained from the teachers, the set of topics was revised and submitted with a complete report of the study to the Dean of the College of Engineering and to a group of the faculty of the Department of Mathematics. Only minor changes were suggested by these individuals. It seems reasonable to assume, therefore, that the following list of minimum mathematical needs of prospective students of the College of Engineering represents a consensus insofar as practicable of the College of Engineering, the Department of Mathematics, and a group of high school teachers of mathematics.

The Minimum Mathematical Needs of Prospective Students in the College of Engineering*

This section lists topics in secondary mathematics, an understanding of which is considered to be indispensable.

It is expected that students who have an understanding of the following topics will be able to begin their mathematics training in college with analytic geometry. These students normally will complete any one of the engineering curricula in four years.

The topics marked with an asterisk are those normally studied in advanced (college) algebra and trigonometry. Students who have an understanding of all topics except those so marked will begin with college algebra and trigonometry as their first mathematics courses in college. These students will probably require four and a half years to complete any one of the engineering curricula.

1. Fundamental operations with integers, common fractions, decimals, and mixed numbers
2. Concept of percentage including per cent of increase and decrease
3. Concept of ratio and proportion
4. Concept of measurement and standard units
5. Expression of a physical magnitude: number and unit
6. Conversion of units in the expressions of physical magnitudes
7. Solution of problems involving physical magnitudes—for example, addition of lengths expressed in feet and inches, calculation of areas, addition or subtraction of angles, etc.
8. Scale drawing
9. Concept of an approximate number, precision of a measurement, significant digits, and rounding
10. Concept of algebraic variables and constants
11. Preparation and interpretation of statistical graphs; i.e. bar, circle, and line
12. Removal of parentheses, brackets, braces, etc.
13. Concept of directed or signed numbers
14. Addition, subtraction, multiplication, and division of signed numbers
15. Addition, subtraction, multiplication, and division of algebraic fractions
16. Addition, subtraction, multiplication, and division of polynomials

* Reproduced from the bulletin "Mathematical Needs of Prospective Students at the College of Engineering of the University of Illinois" published jointly by the College of Engineering and the Bureau of Research and Service of the College of Education of the University of Illinois. Copies may be obtained by writing to the Office of Publications, 358 Administration Building, Urbana, Illinois.

17. Common special products; i.e., $a(a+b)$, $(a \pm b)^2$, $(a+b)(a-b)$, $(a+b)(c+d)$
18. Factoring such expressions as a^2+ab, $a^2 \pm 2ab+b^2$, a^2-b^2, ax^2+bx+c
19. Laws of exponents including negative and fractional exponents
20. Solution of linear equations having numerical and/or literal coefficients
21. Solution of a pair of linear equations including solution by determinants
22. Concept of variation
23. Concept of a function, functional notation, representation of functions by means of statements, tables of related values, graphs, and equations
24. Properties of a linear function—i.e., graphical representation, standard form of a linear equation, the slope and y-intercept of a line
25. Solution of a quadratic equation by factoring, by completing the square, and by the formula
26. Addition, subtraction, multiplication, and division of radicals
27. Addition, subtraction, multiplication, and division of complex numbers
28. The standard form of a quadratic equation, its graph, the nature of the roots, and expressions for the sum and product of the roots
29. The properties of a quadratic function—i.e., graph, intercepts, and maximum or minimum value
30. Solution of a system consisting of a linear and a quadratic equation
31. Solution of pairs of quadratic equations
32. Solution of verbal problems by algebraic methods
33. Solution of equations in which the unknown occurs under a radical sign
34. Binomial theorem with positive integral exponents
35. Scientific notation or standard-form numbers—e.g., 2.54×10^3, 1.2×10^{-4}
36. Computation by means of logarithms
37. Interpolation

*38. Change of the base of logarithms
*39. Solution of exponential and logarithmic equations
*40. The factor theorem
*41. Finding the rational roots of higher degree equations of the form $f(x)=0$ where $f(x)$ is a polynomial in x
*42. Rough sketching of the graphs of higher degree equations
*43. Approximating the irrational roots of higher degree equations, preferably by the method of interpolation
*44. Third-order determinants
*45. Arithmetic progressions
*46. Geometric progressions both finite and infinite

47. Concept of equality, including the symbol, and the postulates of equality
48. Concept of inequality, including the symbol, and the properties of inequality
49. Use of the protractor
50. Use of the compass and straight edge in making simple geometric constructions
51. Concept of a plane angle
52. Concept of a dihedral angle
53. Polygons: triangle, square, parallelogram, trapezoid, hexagon, octagon
54. Circles, including the construction of circles tangent to lines and to each other
55. Angle inscribed in a semi-circle
56. Mensuration of plane figures
57. Concept of congruence
58. Concept of similarity
59. Concept of symmetry
60. Concept of locus
61. Parallelism and perpendicularity of lines
62. Pythagorean theorem
63. Projection
64. Pictorial representation of three dimensions of a plane
65. Parallelism and perpendicularity of lines and planes
66. Parallelism and perpendicularity of planes
67. Polyhedrons: cubes, prisms, pyramids
68. Cylinders, cones, and spheres
69. Concept of a definition, a postulate and a theorem
70. Deductive proof
71. Inductive reasoning: its use in science; and the difference between inductive reasoning and proof
72. Trigonometric functions of an acute angle
73. Values of the functions of 30°, 45°, 60°
74. Solution of right triangles
75. Relationships of acute angles of a right triangle: $\sin(90° - A) = \cos A$, etc.
76. Solution of verbal problems involving right triangles

*77. Definitions of trigonometric functions of any angle
*78. Values of functions of special angles including quadrantal angles
*79. Numerical value of functions of any angle from trigonometric tables; natural and logarithmic
*80. Fundamental trigonometric identities
*81. Addition identities: sin $(A+B)$, etc
*82. Law of sines
*83. Law of cosines
*84. Law of tangents
*85. Area formulas: $K=\frac{1}{2}bc \sin A$; $K=\sqrt{s(s-a)(s-b)(s-c)}$
*86. Solution of oblique triangles
*87. Solution of triangles by logarithms
*88. Radian measure of angles
*89. Graphs of sine and cosine functions
*90. Inverse trigonometric functions
*91. Solution of trigonometric equations
*92. Double-angle identities
*93. Half-angle identities
*94. Proofs of identities
*95. Concept of a vector, a component, and a resultant
*96. Graphical addition and subtraction of vectors
*97. Addition and subtraction of vectors by components

Some topics are not so fundamental as to be classified as indispensable. It is recommended that they be studied if there is time available or by high-ability students whose rate of learning warrants supplementary work.

1. Extraction of square roots by means of the algorithm
2. Slide rule
3. Binomial theorem with fractional and negative exponents
4. Permutations
5. Combinations
6. Probability
7. The inverse, converse, and contra-positive of a statement
8. Polyhedral angles
9. Line values of trigonometric functions
10. Formulas for tangents of the half-angle
11. Multiplication and division of complex numbers in polar form
12. De Moivre's theorem
13. Exponential form of a complex number

How Mathematics Teachers Can Use the List

If there are some students who contemplate entering a college of engineering, the teacher can use this list to direct their study. Since many related fields of scientific study such as physics, chemistry, and mathematics require about the same preparation in high school mathematics, it is expected that there will be many students in high school mathematics classes for whom the list may serve as a guide. There is little reason, however, why all students, irrespective of their needs and interests, should study the same topics. The decision of what is appropriate for the individual students to study rests with the teacher.

In conclusion, the fact that this list of mathematical needs is a *minimum* list deserves to be stressed. It would be indeed unfortunate if the list became the maximum content of a four-year program in mathematics. In order to provide students with a thorough and broad understanding of mathematics it is necessary to go beyond the above list of minimum essentials. It is hoped that each teacher will encourage each student to progress as far as his capabilities will allow.

EDITOR'S NOTE

The names of H. von Baravalle, Lee E. Boyer and Miles C. Hartley were inadvertently omitted from the list of persons (on page 550 of the December, 1951 issue) who had refereed papers for The Mathematics Teacher during the calendar year of 1951.

Can We Teach Pupils to Distinguish the Measurement and Partition Ideas in Division?

By Harold E. Moser
State Teachers College, Towson, Maryland

The question under immediate consideration ought to be studied in the light of number meanings involved and possible dividends to be expected from a slightly different approach to the teaching of the division concept.

It is interesting to note that a few years ago a speaker confronted with the question presently considered easily might have dichotomized the issue by weighing the advantages of a mechanical approach versus the values of meaning. Today, however, the increasing faith in the importance of meaning in the development of quantitative thinking makes it unnecessary to adopt an evangelistic approach to meaning. Almost everybody claims to be teaching meaning and everybody expects a speaker to enumerate its virtues. This unanimity in willingness to witness for meaning does not, however, carry with it the implication that there exists a single, well defined and generally accepted body of meanings basic to a well-rounded program in elementary arithmetic. On the contrary, one finds a very considerable diversity in method, content and materials, all of which are described by their sponsors as making the topic meaningful and sensible to children.

Perhaps a considerable number of the activities currently described as "meaningful" are spurious from the mathematical point of view, but even so one must concede that there are a great many mathematically legitimate approaches to the teaching of meaning. For one thing there is a very considerable body of mathematical principles and generalizations which may be called meanings. Some are broad and basic, such as when a child learns to look at addition as a process for putting together two or more groups. Others are more specific and restricted in application, as the use of the principle of compensation in adding *four* plus *five* plus *six*, wherein one subtracts one from the *six*, adds one to the *four*, and thinks *five* plus *five* plus *five*. Still others take a lesser position in the value hierarchy. An understanding that the addition of two odd numbers always gives an even number may illustrate this lower hierarchy.

The point is that we need not undertake to put everything into the elementary school curriculum that may be described as contributing to meaning. Not all meanings are of equal value. Also, too many relationships presented at one time would make for confusion. It is necessary, therefore, for us to be discriminating in our selection of a core of number ideas to be developed in the elementary school.

Upon what basis shall a selection be made where two or more meaning approaches are possible? Two principles come to mind. It is generally accepted that those number meanings which give structure and form to the number system are of the greatest value to beginners. The same principle applies equally to instruction in the computational processes. Unless it can be shown that the insights and understandings taught help to give structure and system to the computational procedures it may be wiser to postpone the introduction of the concepts to a later spot in the elementary curriculum.

There is one more principle relating to the teaching of meanings that ought to be considered. The meanings taught ought to be found useful to children as mental aids in quantitative thinking. Not all meanings are equally useful. This is not the same as saying that not all meanings are equally important. Some meanings are important

but are not easily adapted to the thought processes of children. An excellent illustration of this point was revealed in the subtraction monograph recently published under the direction of Dr. Brownell.* In this study it was found that the principle of equal additions, taught meaningfully, was not as useful in the thinking of children as was the decomposition principle taught meaningfully. If equal additions is to be taught certainly the principle is important because it gives purpose and sense to the subtraction process. It was found, however, that the principle was neither easily understood nor efficiently handled by third grade children.

In summary, then, two criteria have been offered to serve as guides in the selection of meanings which ought to be taught in the elementary school. (1) They should give sense to the system of symbolization or the computational process being developed. (2) They should be useful to children in their quantitative thinking. They should be of practical assistance in attacking problem-solving situations.

If you accept the principles just proposed for determining the circumstances under which meanings ought to be taught, then we are ready to examine the concepts of measurement and partition division to determine to what extent they satisfy the criteria just offered.

Let us begin with an analysis of the concepts involved. Division may be defined as the process of finding either of two factors when one factor and the product of the two factors are given. In this respect division bears the same relationship to multiplication that subtraction bears to addition. In all practical applications of multiplication one factor is always concrete and the other is abstract, that is to say that a group of a given size is repeated a certain number of times. Two types of division logically arise from this situation since one may divide to find either the missing multiplier or the missing multiplicand. Division in which the quotient expresses the number of times is called *measurement* division, and division in which the quotient expresses the size of each of the like groups is called *partition* division.

Example of measurement division: How many pamphlets, each costing 25¢, can I buy for $2.00?

Example of partition division: If $24 is to be shared equally by six persons, how much money will each receive?

This distinction in the two types of division is not often taught in our schools. Are there good reasons why this differentiation ought to be made? I believe that an affirmative answer to this question is necessary if we apply the criteria cited earlier.

Under the structure principle, it can be seen that the measurement and partitive ideas emphasize the complementary relationships between multiplication and division to the fullest extent. Let us see wherein current practice falls short of that mark. In schools where multiplication is taught meaningfully much emphasis is usually placed upon the principle that multiplication may be viewed as a shortened form of addition. In this approach the size of each of the equal addends becomes the multiplicand and the number of repetitions, or times, becomes the multiplier. This rationalization is especially recommended for young children. It is consistent with the way the multiplication idea appears in the practical affairs of life. This later feature makes for a close correlation between the abstract process and its practical application. Success in problem solving depends upon this transfer.

The development of the multiplication concept does not stop here. At a later point in the elementary school more emphasis is placed on the factor idea. In this step multiplication is carried one step

* Brownell and Others. *Meaningful vs. Mechanical Learning: A Study in Grade III Subtraction.* ("Duke Univ. Studies in Educ." No. 8. Duke Univ. Press, 1949.)

further toward becoming an abstract thought process unencumbered by ideas of collections and aggregates. Thus, 18 is 9 twos, 2 nines, 6 threes or 3 sixes. In time this factor approach will supplant, in considerable measure, the equal addend conception of multiplication for most children. The concept of multiplication, however, will have been represented meaningfully at every level. This is a good program.

The job is not done as effectively in the case of division. At the concrete level, division is currently pictured as finding how many two's, three's, five's, etc., in a number. On the basis of this experience children are expected to generalize that division is the reverse of multiplication. Only one-half of the story has been taught, however, and this half you will recognize as the measurement division concept.

In current practice partition division is generally introduced with the fractional form. To find one-fourth of a number we divide by four. From the pupil's point of view this principle is usually reversed by the teacher in the next grade when he is told that finding one-fourth of a number is multiplication. There is much learning confusion in this area. The relationship between the measurement and partitive ideas on the one hand and multiplication process on the other is never completely tied down. The partitive idea just dangles; it is not represented as an integral part of any process. Some children think of it as associated with fractional operations exclusively. It is exceptional to find an adult who has heard of two meanings for division of integers.

Now let us consider the principle of usefulness in quantitative thinking. The most serious shortcoming of this failure to close the gap between multiplication and division becomes apparent when practical applications of multiplication and division are studied side by side. In a program of meaning arithmetic, children are expected to select the appropriate process to be used in problem solving according to an analysis of the activity described. Thus, if the situation describes putting together equal groups, multiplication is called for. On the other hand, if the activity pictures breaking one large group into equal parts, the situation calls for division. Several examples may be cited:

> Example #1. Multiplication process clearly inferred from a description of the activity: A man set aside \$5 each week for a summer vacation. How much will he have saved in ten weeks?
>
> Example #2. Division process clearly inferred from a description of the activity. How many 5″ stakes can be cut from a strip of wood 20″ long?

There are a good many examples, however, in which the relationship between the activity described and the operation required is not clearly drawn.

> Example #3. I have a bucket that holds two gallons of water and I want to fill a tank that holds 24 gallons. How many buckets of water must I carry?

From personal experience I know that many children have difficulty with examples of this type. The activity is described in terms of putting together equal groups and so children elect to use the multiplication process. There is nothing about the grouping pattern described that suggests division. The division process is used because the multiplication is incomplete. A multiplier must be found capable of giving the desired product.

Examples similar to the last constitute a kind of "twilight zone" between the operations of multiplication and division. Here the logic of determining the process in terms of "putting together" and "taking apart" breaks down. This confusion is less likely to arise where the complete reciprocal relationship between multiplication and division is taught.*

* It is interesting to note that another "twilight zone" is found between the addition and subtraction processes. Consider this problem at

The ease with which children take to the missing term idea in multiplication, and its helpfulness to them in problem-solving situations needs to be studied more completely in actual experimentation. Our work at Towson is still in the preliminary trial stage. We do have evidence to indicate that the approach is especially helpful for problems involving area and volume. With this approach, for example, one principle, $l \times w = A$, will take care of all possible types of examples.

If we accept the desirability of teaching the two meanings of division, what teaching problems may be anticipated? One of the first to come to mind is the lack of a distinctive way for writing symbolically each of the meanings. The conventional algorism is ambiguous on this point. The expression, $6 \div 3$, means simply to find the other factor of six. It makes no difference which of the two factors is missing. The same goes for algorism $3\overline{)6}$. Either algorism answers two questions. "How many groups of three can be made with six?" and also, "If six is divided into three equal groups, how many will there be in each group?" To write each of the division ideas apart from the other will require the creation of special forms.

Dr. Spitzer is the only writer that comes to mind as having discussed this point. In his book, *Teaching Arithmetic*, he suggests that the special form $12 \div 4$ be used for measurement division and that the partition form be written as $\frac{1}{4}$ of 12. I have not been able to accept these suggestions as particularly helpful. Measurement division as a special case exists only when we we are searching for the multiplier (or the number of times). Therefore it seems to me that $n \times 4 = 12$ is the most appropriate way to express measurement division. In a similar manner my present opinion leans to the use of $4 \times n = 12$ as the best way to introduce the partition idea. Skill developed in the use of these horizontal expressions ought to transfer nicely when the time comes to study simple linear equations in one unknown.

What kind of educative experiences are needed to develop these concepts? The answer to this question is still in the making. As mentioned earlier, our Towson experiments are still in the exploratory stage. If the problem is being studied elsewhere there is no mention of the fact in the literature. Most of our experiences have been with children who already have been taught multiplication and division by the usual procedures. We believe these limited explorations are useful and the devices and explanations found effective with these children will eventually become the basis for an experimental try-out which will begin with the initial lessons in multiplication. A tentative program of experiences has been mapped out for each level of learning from the most immature to the use of the higher thought processes.

1. **Manipulatory level.** Activities at this level are entirely concrete. No symbols are used. All attention is focused upon the activity itself as a method for accomplishing a selected purpose. Experiences are built around—
   - (a) Dividing groups into equal parts to find how many there will be in each part.
   - (b) Seeing how many groups of a given size can be made from one large group.
2. **Pre-algorismic level.** First recourse to symbols should be for the purpose of *recording* the nature of the manipulations performed. The pattern of symbolization should correspond very closely with the activity performed, i.e., the pupil should be able to see readily the nature of the grouping in the symbolic record. Repeated subtraction is a most useful way to describe the manipulatory activity associated with division because division represented as repeated subtraction will satisfy the needs of both measurement and partition types of division.
   - (a) In measurement division each subtraction makes one complete group. Count the number of subtractions to find the number of groups.

(*Continued on page 105*)

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the second grade level:

> Joe had four comic books. How many more must he add to his collection to make nine?

When a teacher finds children having difficulty with problems such as these she may find it hard to keep from resorting to cue-word techniques. It is very easy to say, "To find how many more we subtract." A rationalization in terms of the missing addend would seem to be a more satisfying explanation mathematically.

Our Public Relations

By Sister Noel Marie, C.S.J.
College of Saint Rose, Albany, New York

In the future, this period of our history may well be labeled the Era of Public Relations. Dale Carnegie's *How To Win Friends and Influence People* was a tremendous success with individuals and since it was written its psychology has been developed to include groups, businesses, even educational bodies.

We teachers of mathematics appreciate the principles expounded by public relations offices. We vaguely feel that better understandings have developed and that such an office is both necessary and worthwhile. Then in our abstracted manner we work at the problem that has been defying us for so long—public relations are forgotten.

What is the result? What is the rest of our world doing while we are solving our mathematical problems? Dr. C. V. Newsom, Assistant Commissioner of Higher Education in New York State, at a meeting of the Upper New York State Section of the American Mathematical Association, at Syracuse, warned that presidents and deans of colleges are asking why mathematics should be included in college curricula. They feel that it is merely a tool subject and should have no place in general education. We, the teachers of mathematics, have read the literature of mathematics; we are conversant with the two aspects of mathematics, pure and applied; we link mathematics with art, poetry, logic—but we, in our ivory towers keep this knowledge to ourselves. We are neither miserly nor excessively modest about our learning. We just do not consider our public relations.

Again, we are sending out graduates of our liberal arts colleges as high school teachers. How much training are we giving them in being our good-will ambassadors with the next generation? Are we training them as abstract algebraists, as expert statisticians, as elegant geometricians? You cannot shrug off these considerations impatiently. The future of the study of mathematics is involved. We must consider our public relations.

If we were to take a lesson from a public relations office, we would note that one question reiterated is, "How can we service our public?" And more pertinent still, "How shall we tell the public that we are helping them and that we are willing to continue to help them?" Does this sound too materialistic, too closely associated with commerce, too little concerned with the ideals of education *per se?* But this is the age of advertising. Our public is conditioned to the man or the company that sells itself; we teachers of mathematics certainly do not sell ourselves. We have ample opportunity, too, to be of service with the infant science of statistics. So many people who use statistics to prove a point do not know the elements of the theory. As Dr. Newsom points out in the preface to *The Science of Chance* by Horace C. Levinson, " . . . much 'bad statistics' is being promulgated; the public has been victimized in many instances by pseudo-statisticians who draw unjustified conclusions from data."

Dr. Levinson in this book devotes a chapter to the fallacies in statistics. Since the whole "population" is judged by a sample, that sample must be fair, the conclusions valid. Even then the general public misinterprets many findings such as averages (weighted or unweighted), distributions which contain extreme cases, problems which involve too few cases, even the circumstances attendant upon the finding of results. Perhaps it is the fault of the statisticians, or the person who is quoting statistics, that the fact that these are special cases is not emphasized. At least it is an opportunity for the mathe-

matically-minded to point out the weak links in the chain. Automatically when one thinks of errors in interpretation one thinks of the ignoble consequences of the pre-election polling of 1948. Mathematics teachers, surely, should be sufficiently conversant with statistical techniques to emphasize that predictions based on a small sample cannot be entirely accurate. Dr. Levinson points out, too:

> It is not essential to be a technically trained statistician in order to distinguish good statistics from bad; for bad statistics has a way of being very bad indeed, while a good analysis carries with it certain earmarks that can be recognized without great difficulty. In a good job the sources of the data are given, possible causes of error are discussed and deficiencies pointed out; conclusions are stated with caution and limited in scope; if they are numerical conclusions, they are stated in terms of chance, which means that the probabilities of error of various amounts are indicated. In a bad job, there is a minimum of such supplementary discussion and explanation. Conclusions are usually stated as though they were definite and final. In fact, it can be said that a poor job is characterized by an air of ease and simplicity, likely to beguile the unwary reader into a false sense of confidence in its soundness. In this respect statistics is very different from other branches of mathematics, where ease and simplicity are earmarks of soundness and mastery.

On the question of service to the public, what work is more creditable than the rigorous training effected by mathematical discipline? But, judging by the reactions of college freshmen, one doubts that many who have studied algebra and geometry realize that their thinking processes were supposed to have been improved by the courses they had taken. A few examples in class, at the psychological moment, would help to highlight the application of mathematics to life situations, not spread like frosting on a cake, but an integrated development that works out naturally.

The thirteenth Yearbook of the National Council of Teachers of Mathematics is devoted to a description and evaluation of certain procedures used in a senior high school to develop an understanding of the nature of proof. The purpose of the study was "to describe classroom procedures by which geometric proof may be used as a means for cultivating critical and reflective thought. . . . " The data assembled was of two types. The second of these explained the content of the course by which the pupil studied geometric material and the non-geometric to which "transfer" had been made. High-lighted were: the recognition of the need of definitions (problems discussed ranged from school to national situations, with the admission elicited that "it is difficult to agree on definitions and assumptions in situations which cause one to become excited"), inductive proof (editorials provided many illustrations of this type of proof and the pupils were quick to point out its limitations), and assumptions (the Declaration of Independence was analyzed for assumptions, generalizations and conclusions, and modern advertising proved a rich field for illustrations of this type of reasoning). These pupils *knew* that their study of geometry was a study of *method*.

At this point one is reminded that Anthony Standen in *Science Is A Sacred Cow* castigates educators thus, " 'Disciplinary value' is the last thing that a die-hard old fogy of an educator will claim for some study that hangs on for no reason other than senseless tradition." He feels that mathematics should be studied for its own sake; that our schools " . . . by their degraded treatment of mathematics miss a fine opportunity to give some experience of the true." He points out, too, that the "decline of mathematics came about through the modern utilitarian philosophy, which regards the sole purpose of science, of knowledge in general, and of education, as contributing to some 'useful' purpose."

Whether we wish to impress the public with our usefulness (as do the "die-hard old fogies") or with our ability to give some experience of the true (as does Anthony Standen) we should at least be cognizant of the fact that at present we are not very busy about impressing anyone.

(*Continued on page 101*)

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Membership Record

MARY C. ROGERS, *Roosevelt Junior High School, Westfield, New Jersey*

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1.	Denver, Colorado	East High School
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3.	Washington, D. C.	Eliot Junior High School
4.	Washington, D. C.	Gordon Junior High School
5.	Washington, D. C.	McKinley High School
6.	Washington, D. C.	Paul Junior High School
7.	Washington, D. C.	Roosevelt High School
8.	Washington, D. C.	Stuart Junior High School
9.	Washington, D. C.	Western High School
10.	Ocala, Florida	Ocala Junior High School
11.	Lincoln, Nebraska	Lincoln High School
12.	Atlantic City, New Jersey	Brighton Avenue School
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14.	Caldwell, New Jersey	Mt. St. Dominic Academy
15.	Denville, New Jersey	Junior High School
16.	Elizabeth, New Jersey	Battin High School
17.	Englewood, New Jersey	Englewood School for Boys
18.	Glen Ridge, New Jersey	Senior High School
19.	Highland Park, New Jersey	Hamilton Junior High School
20.	Jersey City, New Jersey	State Teachers College
21.	Lawrenceville, New Jersey	The Lawrenceville School
22.	Livingston, New Jersey	Junior High School
23.	Montclair, New Jersey	State Teachers College
24.	New Brunswick, New Jersey	Rutgers Preparatory School
25.	Pennington, New Jersey	The Pennington School
26.	Red Bank, New Jersey	Senior High School
27.	Rockaway, New Jersey	Rockaway High School
28.	South Amboy, New Jersey	Harold G. Hoffman High School
29.	Summit, New Jersey	Kent Place School
30	Sussex, New Jersey	Sussex High School
31.	Swedesboro, New Jersey	Swedesboro High School
32.	Trenton, New Jersey	State Teachers College
33.	Wallington, New Jersey	Wallington High School
34.	Wood-Ridge, New Jersey	Wood-Ridge High School
35.	Rebersburg, Pennsylvania	Miles Township High School
36.	Elizabethtown, Pennsylvania	Elizabethtown College
37.	Richmond, Virginia	John Marshall High School

"All but One" Schools—as of December 1, 1951

1.	Englewood, Colorado	Englewood High School
2.	Allentown, New Jersey	Upper Freehold Township High School
3.	Burlington, New Jersey	Burlington High School
4.	East Rutherford, New Jersey	East Rutherford High School
5.	Glen Rock, New Jersey	Junior High School
6.	Highland Park, New Jersey	Senior High School
7.	Midland Park, New Jersey	Junior High School
8.	Mt. Holly, New Jersey	Regional High School
9.	Newark, New Jersey	State Teachers College
10.	Passaic, New Jersey	Thomas Jefferson Junior High School
11.	Penns Grove, New Jersey	Regional High School
12.	River Edge, New Jersey	Junior High School
13.	Rumson, New Jersey	Rumson High School
14.	Washington, New Jersey	Washington High School
15.	Wildwood, New Jersey	Wildwood High School

Our congratulations to you for your increasingly fine membership record! We have word from National Council headquarters which we are very much pleased to pass on to you. We are certain you will be interested because it is you who has made this good news possible. Mr. Ahrendt writes, "Memberships continue to pour in. At the present time we are just swamped with memberships and orders. We can hardly keep up. The membership is now up to about 8700. We surely should hit

9000 this year, and 10,000 by the end of next year. Our immediate goal is 10,000 members. It would be a *great year* if we could reach that number by May 1952."

We believe you will accomplish this objective. It is an inspiring experience to listen to the enthusiastic reports of State Representatives at our annual meetings. They are achieving most commendable results in this building up of National Council membership. A very great deal of this success is quite evidently due to the steadily increasing cooperation and support given them by their state or local associations and by you, the mathematics teachers from all parts of the country. Your endorsement of National Council services is an invaluable aid. It gives strength and purpose to this entire national structure. National Council is most grateful to you for your interest and support.

Attaining the 10,000 membership goal will mean many more Membership Honor Schools for which National Council certainly plans to give recognition. We shall publish a second listing in the May, 1952 MATHEMATICS TEACHER if you wish us to do so. But we do need your assistance in locating these schools. If your school qualifies for this recognition—if you know of a neighboring school which should be listed—won't you please write us about it. Simply complete the following form and mail it to

Mary C. Rogers
307 Prospect Street
Westfield, New Jersey

We shall need this information by March 1, 1952, for including in the May, 1952 report. We shall look forward to hearing from many of you. Thank you very much for this assistance.

MEMBERSHIP REPORT TO THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

School ______________________________

School Address ______________________________

Numbers of Teachers of Mathematics in Your School ______________________________

Number of National Council Members ______________________________

Names of Individual Members ______________________________

____

Report submitted by ______________________________ ______________________________

Your Address ______________________________

Our Public Relations

(*Continued from page 99*)

One aspect of mathematics that is realized and appreciated in general only by the mathematician, is that of mathematics as an art—as one of the humanities. Witness the quick smile of the librarian when you ask for David Eugene Smith's "The Poetry of Mathematics." Obviously she considers the words "poetry" and "mathematics" as being incongruous. Moreover, her smile reflects the attitude of the general public. "Mathematics," you will be told, "has to do with numbers. Poetry needs imagination—inspiration." Dr. Smith agrees with them. But, he claims, poetry and mathematics are "equals in the display of the results of man's imagination." He feels, too, that "the strongest bond of sympathy between mathematics and poetry . . . is the endless invention of each." In his book, he first considers poetry in mathematics, quoting various writers' opinions on the subject, including Weierstrass's "a mathematician who is not somewhat of a poet will never be a perfect mathematician." Then he talks about mathematics in poetry citing the mathematical precision of the sonnets. Finally, he unites poetry and mathematics, showing, as mentioned above, that both are works of the imagination. The points of contact between the two may be found in the conciseness of expression employed by each, the fact that each is seeking for the truth, the consistency of each, and finally, the "endless invention of each." This essay of Dr. Smith's, as well as "The Call of Mathematics" was written many years ago. Yet the reminder he gave then, "In every generation men have arisen to question its value . . . " is more than ever applicable today.

(*Continued on page 156*)

APPLICATIONS

Edited by Sheldon S. Myers
Department of Education, Ohio State University, Columbus, Ohio

Ar. 13 Gr. 7–9 *Elements and Minerals of the Earth*

Here is a guidesheet involving the use of per cent developed by John Kinsella at the University School, Ohio State University, in 1943.

1. Man knows of 92 elements on the earth like iron, copper, lead, oxygen, helium, aluminum and so on. (This statement is still true today and the percentages below are still valid since the recently discovered elements above 92 do not occur naturally on the earth.—S.S.M.)
2. An element cannot be chemically separated into parts other than itself, whereas a compound can be separated chemically into elements or other compounds.
3. Of the 92 elements only a few are important in minerals and rocks. Thousands of samples have been analyzed into percentage tables. These tables include all the elements, yet only 8 of them are necessary to make up 98% of the surface or "crust" of the earth.
4. These are:

Oxygen	46.46%
Silicon	27.61%
Aluminum	8.07%
Iron	5.06%
Calcium	3.64%
Sodium	2.75%
Potassium	2.58%
Magnesium	2.07%
Total	?%

5. The remaining 84 elements would total ________% in order to account for the whole of the earth's surface.
6. Common sand (silicon dioxide) consists of 28.3 pounds of silicon to every 32 pounds of oxygen. What per cent of sand is silicon? ________ What per cent of sand is oxygen? ________
7. Common limestone, called calcium carbonate, consists of 40 pounds of calcium, to every 12 pounds of carbon, to every 48 pounds of oxygen. In table form this is:

Calcium	40 pounds
Carbon	12 pounds
Oxygen	48 pounds
Total	? pounds

   This means that in every 100 pounds of limestone, ________ per cent is calcium, ________ per cent is carbon, and ________ per cent is oxygen.
8. If you would like to study the per cent composition of more chemical compounds, ask your instructor for more examples. (See Lange's "Handbook of Chemistry," pages 66–109, for a table of minerals and their composition.)

Ar. 14 Gr. 8–12 *An Improvement in Standard Notation*

The following letter from J. W. Clegg of Battelle Memorial Institute, Columbus, Ohio, proposes an improvement in the standard notation now taught for very large or small numbers. The letter was also sent to *Chemical Engineering Progress* and printed in the January, 1951 issue.

Sir:

In the physical sciences it has long been customary, for the representation of very large or very small numbers, to use a number of a magnitude between 1 and 10 multiplied by 10 to the appropriate power, as 6.06×10^{23}, or 1.84×10^{-5}. Not only is this notation cumbersome, but it also uses the awkward "×" to indicate multiplication and presents the opportunity for its confusion with "x" standing for an unknown quantity.

To overcome these objections, the writer pro-

poses an exponential notation in which the power of ten by which the decimal number is to be multiplied is placed as a superscript over the decimal point, thus: $6\overset{23}{.}06$ and $1\overset{-5}{.}84$. This notation is logical and concise. It may be as readily set up in type, employed for typewritten material, or written by hand, as the older notation with "×." The superscript may conveniently be regarded as the number of places the decimal point must be moved to the right, if the exponent is positive, or to the left, if negative.

From its primary use for very large and very small numbers, the proposed notation may be conveniently extended to express all factors in a complex expression as decimal numbers between 1 and 10, with the appropriate exponent added. This furnishes a ready grasp of the order of magnitude of the quantity and is particularly useful for computations with the slide rule or with logarithms when only the mantissas are employed.

As an example in a common chemical engineering calculation, the Reynolds number for 4800 lb./hr. of water at 70° F. flowing in a 1.049-inch diameter pipe is computed:

$$\mathrm{Re}=\frac{DV\rho}{\mu}$$

D = diameter in feet
V = velocity in feet per second
ρ = density in pounds per cubic foot
μ = viscosity in pounds per foot-second

$$\mathrm{Re}=\frac{1.049}{12}\ \frac{4800}{(62.30)(3600)(0.00600)}\ (62.30)\ \frac{1}{(0.9787)(0.000672)}.$$

In the proposed notation, this becomes:

$$\mathrm{Re}=\frac{(1.049)(4\overset{3}{.}80)}{(1\overset{1}{.}2)(3\overset{3}{.}60)(6\overset{-3}{.}00)(9\overset{-1}{.}787)(6\overset{-4}{.}72)}.$$

Ignoring the exponents temporarily, the magnitude of this quantity may be estimated to be about 5/1600, or 0.003. Summing exponents according to the usual rules, the approximation becomes $0\overset{7}{.}003$. The slide rule figure is 2954, and it is apparent that this must be $0\overset{7}{.}002954$, or 29,540.

Many similar applications immediately become apparent.

J. W. Clegg
Columbus, Ohio

There are many examples of large and small numbers of intense interest to junior high pupils which can be used to give practice on this level in the use of standard notation or of the new notation proposed in the above letter. Following are some appropriate ones:

Re-write the numbers below in the shorter ways:

Description	*Number*	*Standard Form*	*New Form*
Earth to Moon	740,000 mi.	7.4×10^5	$7\overset{5}{.}4$
Diam. of nitrogen molecule	.000000018 cm.	1.8×10^{-8}	$1\overset{-8}{.}8$
Earth to sun	93,000,000 mi.	9.3×10^7	$9\overset{7}{.}3$
Earth to farthest known star in 1934	8,225,000,000,000,000,000 mi.	______	______
Mars to Sun	141,500,000 mi.	______	______
Diam. of Sun	864,000 mi.	______	______
Diam. of solar system	100,000,000,000 mi.	______	______
Diam. of our galaxy	10,000,000,000,000,000,000 mi.	______	______
Weight of hydrogen proton	.0000000000000000000000016608 grams	______	______
Molecules in one cc. of air at sea level pressure and 0° C. temp.	27,000,000,000,000,000,000 mol. per cc.	______	______

Ar. 15 Gr. 7–9 *Speeds*

Children enjoy the topic of speeds and like to make comparisons of speeds. In the February, 1951 Mathematics Teacher, this department presented a set of problems involving animal and bird speeds. The following two sets of problems on speeds were together on a guidesheet developed by Dr. Charles C. Weidemann in 1937 at the University School, Ohio State University.

Travel on Wheels

The first wheels were solid discs. The spoked wheel was developed to assist travel over rough roads with heavy loads. The iron rim of a slightly dished wheel increased its durability. Greater weights caused a development from two to four-wheeled vehicles. The invention of the wheel made possible much of the modern achievements in industry, transportation and construction. From chariots, to stages,

to cabs, to buggies, wagons and automobiles is a quick and partial survey of the development of travel on wheels.

Because the roads were in bad shape, wagon travel was rarely more than 15 miles per day. On an 8-hour day, this was slightly less than ________ miles per hour. "Flying wagons" from Manchester to London traveled around 150 miles in 100 hours or about ________ miles per hour. Faster stage coaches required about 30 hours between Liverpool and London, a distance of about 180 miles. This was ________ miles per hour. With better roads and horses as many as 100 miles in 12 hours became possible. This is________ miles per hour.

In the United States one of the famous horse-drawn stage routes extended from St. Joseph, Missouri, to Sacramento, California. Twenty-five days of 10 hours each were required to travel about 2300 miles. This was about ________ miles per day and ________ miles per hour.

Various Athletic Rates of Performance

1. Medica swam at the rate of 2.62 miles per hour.
2. Soulding walked a mile in 6 minutes 25.8 seconds, or about ________ m.p.h.
3. Cunningham ran a mile at the rate of 14.59 miles per hour. The time for this mile was ________.
4. Owens dashed 100 yards in 9.4 seconds or ________ miles per hour.
5. Ski jumping is between 60 and 100 miles per hour.
6. An auto-paced bicycle traveled 1.271 miles in one minute or ________ m.p.h.
7. Tilden drove a tennis ball at the rate of 118 miles per hour or ______ feet per second.
8. Sarazen drove a golf ball at the rate of 2 miles per minute or ________ m.p.h.
9. A pitched baseball was measured at 127 miles per hour.
10. A boxing blow traveling a distance of 10 inches was rated at 135 miles per hour or ________ feet per second.

Ar. 16 Gr. 7–9 *Rain and Water Power*

It is to be hoped that this department will inspire teachers of mathematics with a vision of the vast and unlimited possibilities of their subject. These applications are suggestive of what creative imagination on the part of the mathematics teacher can do in the way of enrichment, motivation, and practice. Surely we as teachers should give full consideration to those applications which bring wonderment, interest, enthusiasm, and a desire to use mathematics to our youngsters. Following is a guidesheet by John Kinsella which should interest young and old alike.

Rain and Water Power

1. The rain and snow falling on the earth each year would fill a lake 1,000 miles long, 300 miles wide, and about 480 feet deep. How many cubic miles is this? ________ Lake Superior is about 1/11 of this volume or about ________ cubic miles.
2. While water travels, it works. Some of this is used by man; most of this work goes into changing hills, prairies, and valleys. In the United States, the streams and rivers from mountains to oceans develop around 419,000,000 horsepower. Write the name of this number. ________________
3. Rivers moving at the rate of 2 miles per hour will move pebbles as large as a hen's egg. It has been said that the *transporting power of a current increases with the sixth power of its speed.* This means that the *transporting power* of a stream traveling 4 miles per hour is 64 times as great as a stream traveling 2 miles per hour; and a stream traveling 6 miles per hour has a power to transport stones 729 times as great as a stream traveling 2 miles per hour. Fill in the table on the opposite page.
4. The question arises how much weight can a stream move? Beds of small fast-flowing mountain streams contain boulders weighing from 50 to 500 pounds, or an average of ________

Stream flow mi./hr.	*Multiple of 2 miles/hour*	*Sixth power of multiple*	*Power to transport in terms of 2 mi./hr.*
2	1	$1\times1\times1\times1\times1\times1$	1
4	2	$2\times2\times2\times2\times2\times2$	64
6	3	$3\times3\times3\times3\times3\times3$	729
8	4	$4\times4\times4\times4\times4\times4$	?
10	5	?	?

pounds. Streams in the Rocky Mountains have rolled boulders weighing from 9 to 13 tons. This is from ________ to ________ pounds. When St. Francis Dam near Los Angeles broke, blocks of concrete weighing as much as 20,000,000 pounds were moved. This is ________ tons.

5. Another way to look at this problem is to study the material that the Mississippi River carries to the Gulf of Mexico. They estimate that the river carries in one year:

136,000,000 tons of material dissolved in the water
384,500,000 tons as fine mud and silt
56,500,000 tons by rolling pebbles along the bottom

? total tons; write the name of this number ________

223,000,000 tons carried by other rivers of U.S. to the sea

? grand total tons; write the name of this number ________

Departmental Editor's Note:

The following letter should be of interest to our readers.

Dear Mr. Myers:

The members of the Mathematics Department of The Manlius School have noted with interest your article in the December issue of THE MATHEMATICS TEACHER relating to applications of a military nature. Are you familiar with the book, *Military Applications of Mathematics* by Paul P. Hanson, McGraw-Hill, 1944, 447 pages?

To my knowledge this reference contains the best organized work on this subject and has been used by many as a reference book and has seen some use as a text. Mr. Hanson has been a member of the Mathematics staff of The Manlius School since 1943.

The book is now out of print but is undoubtedly available in many libraries.

JOHN W. MACDONALD
Head of the Mathematics Department
The Manlius School
Manlius, New York

Can We Teach Pupils?

(*Continued from page 97*)

(b) In partition division each subtraction places one object in each of the equal groups. Example: If 12 is to be arranged evenly into 4 groups, how many will there be in each of the equal groups? The first four subtracted from twelve will place one unit in each of the four groups required. Each subsequent subtraction will increase the size of each of the equal groups by one. The total number of subtractions will determine how many objects there will be in each of the equal groups.

3. Algorismic level. The division algorism will be presented as a short way to write the work performed by subtraction. (Again there is the exact parallel with the procedures used in teaching multiplication.) At this stage the pupil is carried further from the task of actual grouping toward an attack based upon an analysis of number relationships. In the first lessons at this level the children may speak of finding the missing multiplier or the missing multiplicand, but gradually the emphasis will change to finding the missing term. In this way multiplication and division will be found to be two ways of writing relationships involving the same set of factors. This is the goal that all of us have been working toward for years, albeit with not too conspicuous success. It is time to experiment with a different approach to see whether the efficiency cannot be improved.

No apologies are offered, in conclusion, for describing a project so incompletely explored experimentally. The primary purpose of this paper has been to interest teachers in thinking about an area which seems to have profited less from the change from mechanical to meaning theory than any of the other operations with whole numbers. There is reason to believe that a better job can be done than we now do and it is my personal opinion that teaching children to distinguish between the two kinds of division is a good way to begin.

DEVICES FOR A MATHEMATICS LABORATORY

Edited by EMIL J. BERGER
Monroe High School, St. Paul, Minnesota

This section is being published as an avenue through which teachers of mathematics can share favorite learning aids. Readers are requested to send in descriptions and drawings of devices which they have found particularly helpful in their teaching experience. Send all communications concerning **Devices for a Mathematics Laboratory** to Emil J. Berger, Monroe High School, St. Paul, Minnesota.

COST+OVERHEAD+PROFIT=SELLING PRICE

Students often fail to grasp the relationship between selling price, cost, overhead, margin, and profit. A helpful teaching device which is easily constructed is pictured in Figure 1.

To produce the device procure a piece of wood approximately 18″ long, 1″ wide, and 1″ thick. Cut this piece of wood into three sections of lengths 11″, 4″, and 3″ respectively. Drill $\frac{1}{4}''$ holes $1\frac{1}{2}''$ deep into one end of the 11″ section, into both ends of the 4″ section, and into one end of the 3″ section. Glue a dowel pin into the hole at the end of the 11″ section and another into one end of the 4″ section. Trim these pins off so that they will protrude $1\frac{1}{2}''$ beyond the stocks of the 11″ and 4″ sections.

Assemble the pieces as pictured. Paint one entire side bright green and spread the words SELLING PRICE along the whole length in black lettering.

Paint the second side yellow. Print the word COST on the longest section and spread the word MARGIN over the two remaining pieces. (This side is not visible in the picture.)

Paint a third side orange and again label the longest section COST. Print the word OVERHEAD on the middle-sized section and the word PROFIT on the shortest section.

When students assemble and disassemble the device they are able to see the relationships clearly.

The device shown in the picture was produced by one of the students in a mathematics class at Millersville State Teachers College, Millersville, Pennsylvania.

GEORGE R. ANDERSON
State Teachers College
Millersville, Pennsylvania

THREE DIMENSIONAL GRAPHS

The building of three dimensional graphs is an activity that will help students extend their appreciation and under-

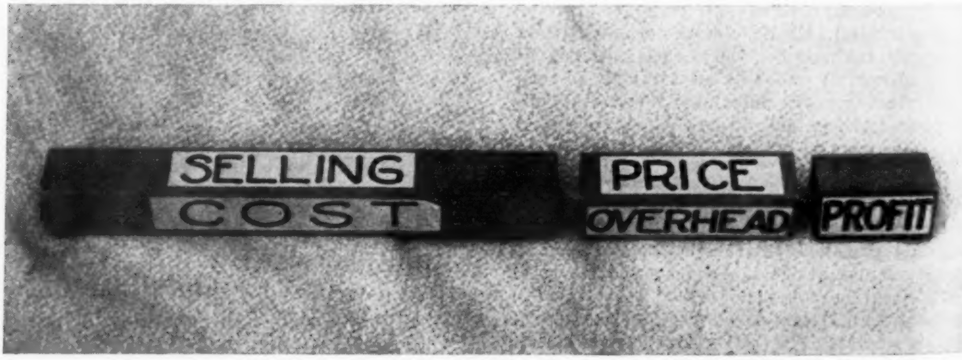

FIG. 1

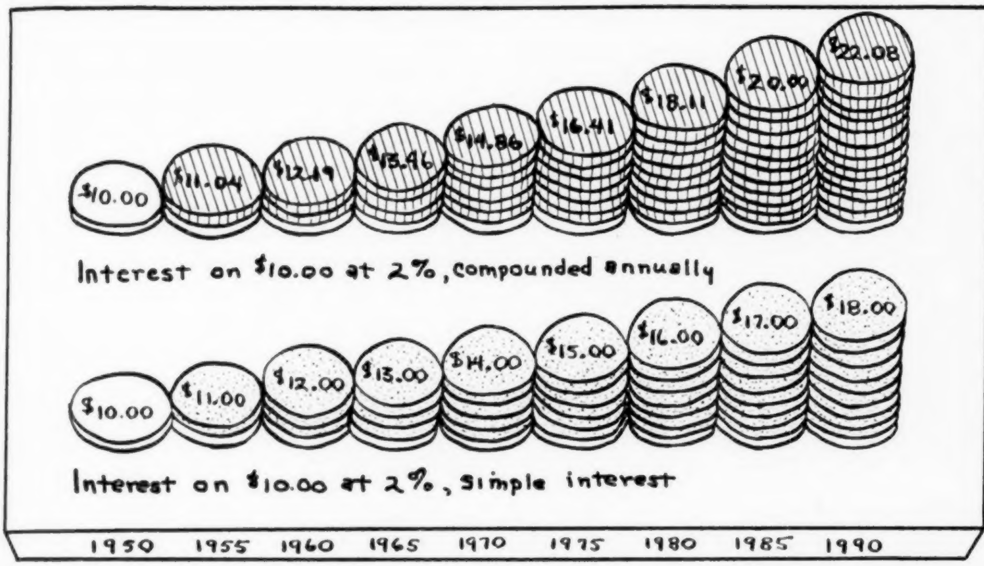

FIG. 2

standing of some rather unique methods that may be employed in representing data and numerical relations concretely. The three dimensional analogues of the usual two dimensional pictographs, bar graphs, and line graphs may, for the sake of convenience, be called object graphs, stereographs, and surface graphs.

Materials used to represent quantities in the object graph include such articles as coins, blocks, bottle caps, poker chips, spools, and small plastic models or toys. Stereographs are actually extensions of two dimensional bar graphs. By employing third dimensional markers such as dowels, balsa strips, or wooden blocks it is possible to illustrate graphically how three classes of data are related instead of two. (See Figures 3 and 4.) In order to graph surfaces it is necessary to set up reference lines in space; in the rectangular coordinate system this means three mutually perpendicular axes. A description of the device pictured in Figure 6 will help clarify how this may be accomplished.

The object graph illustrated in Figure 2 shows how money invested at compound interest grows at a rate far exceeding that invested at simple interest. The graph in the drawing is made of poker chips which are glued together and mounted on a wooden base. The front row of chips shows the amounts accumulated at simple interest according to the law, $A = p(1+rt)$, where A is the amount, t is the time in years, p is the original investment, and r is the rate. The time t is represented along the lower edge of the board, and the amounts which accumulate over different periods of time are indicated by the heights of the different stacks of chips. The white chips along the bottom represent the original investment which in the case of this particular graph is \$10.00. If r is assumed to be 2% per annum, then the blue chips (lightly shaded) stacked on top of the white ones represent the simple interest. The scale of representation for the blue chips is one chip per dollar correct to the nearest dollar.

The back row of chips illustrates how the amount grows when the interest on \$10.00 invested at 2% is compounded. The white chips on the bottom again represent the original investment. The red chips (shaded dark) represent the increases in the amount due to the earning power of interest that is compounded.

The reader will note that the graph has only two reference lines—a horizontal

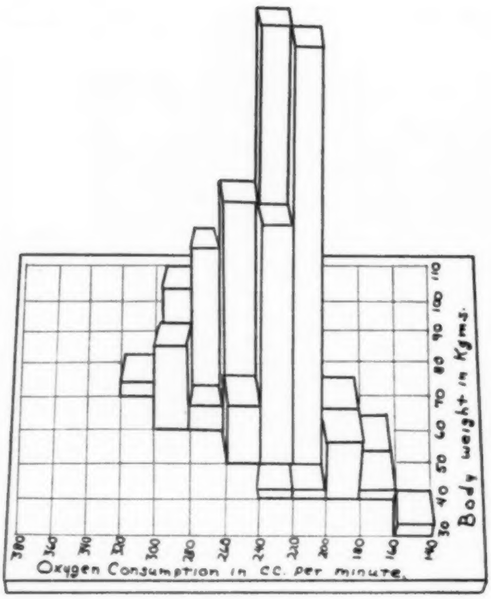

Fig. 3

axis representing time and a vertical one representing amounts. An extension of this graph could be effected by letting the left-hand edge of the board represent either varying rates of interest or different sized original investments and then completing additional sets of double rows in the manner described above. Similar graphs using plastic cars, aeroplanes, people, animals, tools, buildings, etc. will add interest and vividness to representations of other kinds of information.

As stated before the graph in Figure 3 is a stereograph. This particular graph shows the frequency of persons with various oxygen requirements related to the weights of the individuals. The horizontal axis along the lower edge of the base represents oxygen consumption in cubic centimeters, and the axis along the right-hand edge of the base represents body weight in kilograms. Each vertical wooden block represents the population frequency of persons with the same oxygen requirement and the same body weight. The scale of representation is $\frac{1}{2}''$ per one million population. A similar graph could be constructed showing the frequency of pupils with the same height and weight.

Figure 4 illustrates another use of stereographs. The horizontal plane is a two dimensional bar graph comparing the average daily temperature for Minneapolis during the month of February, 1951. This part of the stereograph is shown in Figure 5. A temperature scale is located along the left-hand edge and the days of the month are numbered along the lower edge. The temperature scale ranges from $-16°$ F. to $+40°$ F.

By turning again to Figure 4 the reader can see that there are dowels located both

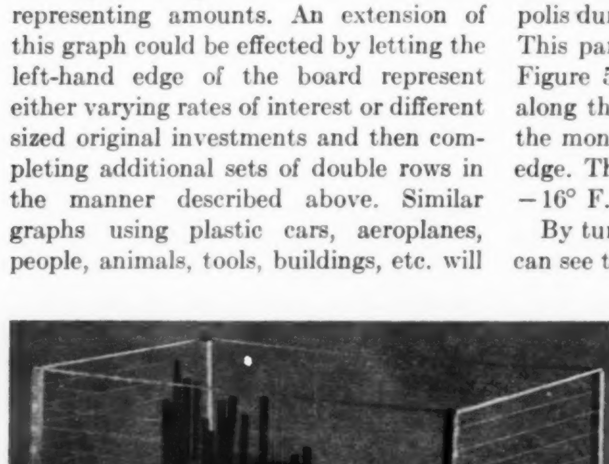

Fig 4

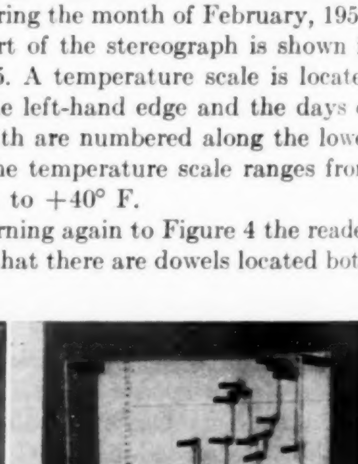

Fig. 5

below and above the horizontal plane. Some of them pass through the plane; others are terminated by the plane. The part of any dowel which extends below the plane is painted white, while that section which appears above the plane is painted a dark color. An inspection of the figure will reveal that the dowels vary in length, and that there is one for each day

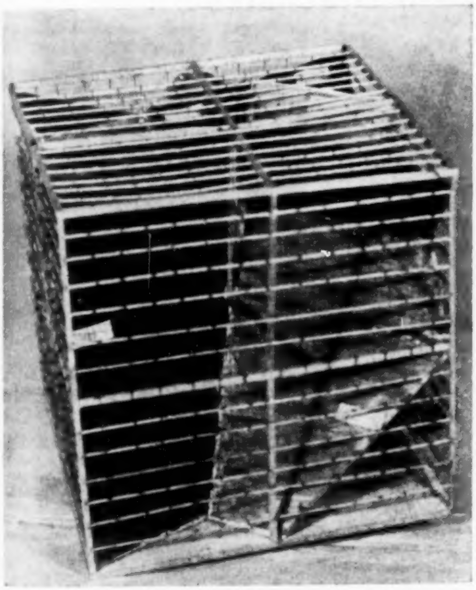

Fig. 6

of the month. With this arrangement it is possible to compare the extremes in temperature above and below the average for the month which was $+17°$ F. The horizontal plane is perpendicular to the vertical reference lines at this reading. The range of the vertical reference frame is from $-27°$ F. to $+50°$ F. Strings are strung around the vertical framework at 5-degree intervals and parallel to the horizontal or "average" plane. These are helpful in comparing daily extremes with the average. For some days the dowels are visible on both sides of the horizontal plane. This means that the temperature on those days went both below and above the average for the month.

Figure 6 is an illustration of a method of graphing the equations of three planes in space. The three planes are independent and simultaneous; that is, they are not coincident, but they do have a common solution. The common solution for the three planes which are illustrated is a point; this means that they have one and only one point in common. The equations of the planes are as follows:

(1) $x-2y+z=0$
(2) $x+\ y-z=0$
(3) $x+\ y+z=12.$

In the picture the plane of the first equation is visible at the lower left; the plane of the second is visible at the lower right, and the plane of the third may be seen at the top.

The basic elements of the reference frame on which the equations are graphed are three mutually perpendicular axes—an x-axis, a y-axis, and a z-axis. The last two are visible in the picture "inside" the box-like structure. The y-axis runs from front to back and is perpendicular to the box face which is toward the observer at the point where the two heavy wooden sticks intersect at right angles. The z-axis is the vertical axis; it is perpendicular to the plane of the top box face. The x-axis is perpendicular to the y and z axes at their intersection, but it is not visible in the picture; it is "behind" the planes.

The entire structure is subdivided into little cubes which are outlined by strings. These strings are fastened to the outer framework of the box at the points which appear as little dark spots on the wooden strips. This arrangement makes it possible to locate points in space with comparative ease. Planes are graphed by locating three non-collinear points on each. This model is extremely useful for giving meaning to the solution of three equations in three unknowns.

Donovan A. Johnson
University of Minnesota
Minneapolis, Minnesota

REFERENCES FOR TEACHERS

Edited by WILLIAM L. SCHAAF
Department of Education, Brooklyn College, Brooklyn, N. Y.

Modern Calculating Machines

IT IS a far cry from Pascal and Leibnitz to the giant "mechanical brains" of today whose dramatic development during the past decade has been literally phenomenal. Time and again, newspapers and magazines have hailed them enthusiastically. When highly technical equipment for solving differential equations becomes front-page news, then mathematics indeed belongs to the celebrated man in the street.

In general, there are two major types of modern computing machines: the *digital* type and the *analog* type. A digital computer deals directly with the digits of the original problem, and the solution may be obtained to as many decimal places as desired. The familiar office comptometer and desk calculator are examples of digital machines; huge modern machines, however, may fill a large room, and contain thousands of moving parts or electron tubes. The analog computer, on the other hand, first translates the numerical data of a problem into other analogous quantities, such as length, or speed, or voltage. When the solution has been obtained, it must be translated back in terms of the original data; the solution is always approximate. Familiar examples of analog type devices are the slide rule and the planimeter. Among the giant analog machines of today we recognize (1) differential analyzers for solving differential equations; (2) harmonic analyzers and synthesizers for studying wave motion; (3) network analyzers for problems concerning power circuits; and (4) machines for solving various types of algebraic equations.

In these bibliographic notes we have excluded for the most part extremely technical discussions as well as very brief articles and "over-popularized" accounts.

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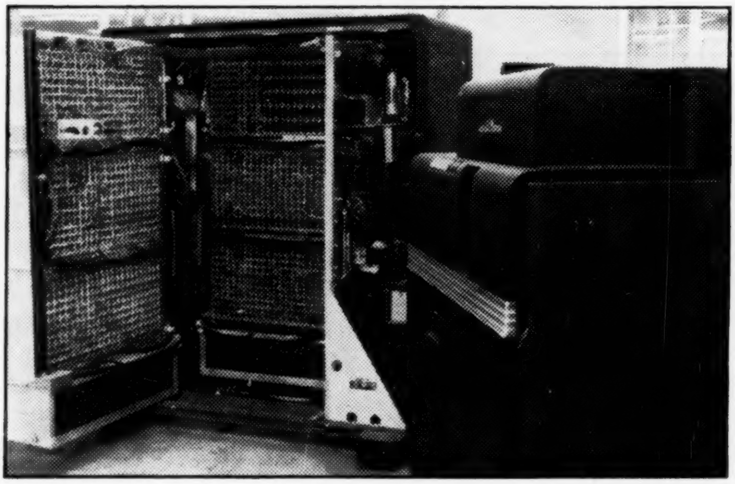
Big Electronic Computers Speed Automotive Research and Testing

With the help of new brain-like machines, automotive engineers are now solving complex research and production problems in record time.

Popularly dubbed "mechanical brains" these uncanny devices are taking much of the time and labor out of mathematical analysis in the research laboratory. About equivalent in size to three home refrigerators combined, they are big brothers of the common desk calculators that have served business, industry and science for many years.

Although its performance bears a startling resemblance to human intelligence, and in fact makes man's powers of calculation seem puny by comparison, the electronic computer's own creators object to the term "mechanical brain."

They point out that while one of these com-



Out of this electronic "brain" come answers to problems in automotive engineering.

plex "nerve systems" of electric circuits and tubes is a whiz with figures, it has no real intelligence, cannot reason or set up propositions and theories, and must be fed specific problems that men devise.

In the opinion of Professor Howard Aiken of Harvard's Computation Laboratory, computers "can't think any more than a stone; they're time-saving tools; pure and simple."

But as tools, technicians believe, the potentialities of these modern computing devices are nearly boundless. Far from being mere instruments of academic mathematics, they are proving their practical worth in many ways.

While only recently made available to industry, automotive, aircraft and other types of manufacturing concerns already are putting the computers to work in research laboratories and statistical departments. For a longer time, huge prototype models have served as invaluable aids to science, especially in the fields of nuclear physics and astronomy.

With such rapid-fire computers, it is now possible to analyze quantities of data and work out involved equations that would completely overwhelm old style computation methods. An hour's output by the type of machine now being used in the automotive industry, is about equivalent to what a mathematician could do in 10 years, armed only with pencil and paper.

As an example of the time and human effort these electronic wizards can save, one automotive company recently used a large automatic computer to determine the cause of vibrations in a vital part of a new model car being developed. It took eight weeks to solve the problem, but engineers on the job estimated that 100,000 man hours would have been spent to get the same answer with ordinary desk calculators.

Another company calculates main bearing loads on one of the big computers, which in three or four days supplies answers that an engineer formerly spent several months to find. In another case, a company built its own special computer for studying crank shaft vibrations. It solves a problem in five minutes that once took six to nine hours.

By doing the laborious arithmetic in highly involved mathematical analysis, which otherwise would be impractical or impossible, it is predicted that computers may eliminate much if not all time-consuming, costly trial and error experimentation.

A leading mechanical engineer in the automotive industry explains how this can be achieved:

"In mechanical engineering, for example, the effects of mass and stiffness, of vibration and stress are well known. If a new design does not lead to radically different operating conditions, we know which of these factors determine the action of the assembly.

"In this field, an enormous amount of work has been done and it is now possible to predict the action of very complicated mechanical parts and processes by mathematical calculations before actual experimental models are built. In order to design the lightest and cheapest installations, however, several possible layouts with many variations of mass and stiffness should be tried. Consequently, the mathematical study takes a great deal of time. Here is a field where fast, modern calculating devices are becoming very useful."

There are many jobs in the industry where electronic computers can be applied, such as:

Studying properties of materials such as strength and durability.

Automatically recording data in testing engines.

Studying heat flow problems in engines.

Designing the shape of gear teeth and the setting of regulators on tooth cutting machines.

Determining the optimum contours and timing of engine cams.

Working out time-consuming details in tooling up for new models.—*Automobile Facts*, Vol. X (September 1951), p. 2.

MATHEMATICAL RECREATIONS

Edited by Aaron Bakst
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The problem of teaching quadratic equations is always plagued by a dearth of motivating materials. Usually, in desperation, we are prone to resort to an admission that quadratic equations rarely offer an opportunity of presenting and utilizing "real" situations. We shall not dwell here on the philosophical discussion concerning the "realness" of a mathematical problem. Suffice it to say that the *realness* of a problem situation is entirely dependent on a major criterion of the pupil's self-identification with the given situation. If the pupil cannot discern or perceive a situation which may actually take place within his own experience, then there is no hope of the problem situation being real. Unfortunately, there persists a notion that unless we have some number problems or problems which are associated with the law of falling bodies, or some camouflaged maxima and minima problems concerning the areas and perimeters of rectangles, we might resign ourselves to the fact that quadratic equations might be taught just for the sake of factoring and the use of the formula. The way out is easy and so conveniently simple. This department proposes to shatter this illusion. . . .

Suppose that the topic on quadratic equations is taught on Monday morning. Also let us suppose that there was a school dance on the preceding Saturday night. What is there more real than a school dance? But how easily this affair lends itself to the introduction of quadratic equations.

The teacher may pose two questions for the consideration of the class:

Question 1. If there were 25 pupils at the dance and all those present danced, but nobody had the same partner more than once (assuming that the sex of the partner is not taken into consideration), how many couples danced during the entire evening?

Question 2. The proceeds from the dance were to go to the G. O., and each couple was charged 5 cents per dance. The total amount collected was $9.50. How many pupils attended the dance? [The assumption is that all danced, that no one had the same partner more than once, and that there was not "cutting-in" or "changing partners."]

If there were 25 pupils at the dance, then John, for example, had 24 partners during the evening. He could not dance with himself. Thus, every pupil had 24 partners, and we ought to conclude that the total number of dancing couples was $25 \cdot 24 = 600$. But, if John danced with Mary, then Mary could not dance with John again. Thus, we must make a correction for the "repetitions" of partners. The product must be divided by 2. In other words, the total number of dancing couples was 300.

This result leads us to a generalization. Suppose that there were n pupils at the dance. Then each pupil danced with $(n-1)$ partners once and once only. And the total number of couples that danced was $n(n-1)/2$.

Now we are ready to answer the second question. If $9.50 was collected from the dancing couples (regardless who paid), and if each couple was charged 5 cents per

dance, then there were 190 dancing couples (and this does not require any higher arithmetic processes than simple division in order to arrive at the result). Then we set up the equation

$$\frac{n(n-1)}{2}=190,$$

or

$$n^2-n-380=0.$$

The roots of this equation are either 20 or -19. We discard the negative root for the obvious reason that there could not be a negative number of couples (and this should lead to a timely discussion of the meaning of a negative number in such a situation). Thus, there were 20 pupils at the dance.

If the use of a Saturday night dance may be too much for a staid school, then the teacher may substitute "hand shakes." Or he may use any other type of example in which a game is played by two persons only.

The introduction of the notion of maxima and minima in an algebra course (without resorting to the methods of calculus) requires the exercise of extreme caution on the part of the teacher. Although such problems might seem to be very simple, they are generally difficult from the conceptual point of view. In order to overcome such difficulties the teacher must develop the fundamental notion of variability. Let us consider the following example.

Given some number n. (It may be a number, or it may represent the numerical value of the length of some straight line segment.) This number must be divided into two parts so that their product is a maximum. (In the case of the straight line segment, the two parts form the sides of a rectangle so that the area of the rectangle is a maximum.)

Let the number n be divided into two parts: x and $(n-x)$. Then the product is

$$(n-x)x=M.$$

Then

$$x^2-nx+M=0,$$

and

$$x=\frac{n\pm\sqrt{n^2-4M}}{2}\cdot$$

In order that the value of x be real the expression under the square root must be positive. In order that the value of $4M$ be the greatest it must not be less than n^2 (it cannot be greater than n^2). Thus, $4M=n^2$. Then

$$x=\frac{n}{2}\cdot$$

(The required area will then be a square.)

Once the technique of examining for maximum (or minimum) is thus established a whole series of recreational problems may become available for classroom use. We shall cite some of them.

A kite is made in the shape of a sector of a circle of a given radius R. Suppose that the perimeter of the kite is given. What should be the length of the arc of the circle?

At what distance from the surface of a table should we suspend a light bulb so that it will give the greatest illumination of an object which is not located directly underneath the light bulb?

This department will publish problems on quadratic equations which the readers are invited to submit and which illustrate the principles and ideas indicated above.

High School Mathematics Letter

A mathematical letter will be mailed twice a semester to high school teachers by the University of Oklahoma chapter of Pi Mu Epsilon. Each letter will contain mathematical news, a short article and a problem section. High school students are invited to submit solutions to the problems. A list of correct solvers will be carried in the next letter. Teachers wishing to receive these letters should send their name and school address to Professor Richard Andree, Department of Mathematics, University of Oklahoma, Norman, Oklahoma.

RESEARCH IN MATHEMATICS EDUCATION

Edited by John J. Kinsella
School of Education, New York University, New York 3, N.Y.

The Question: Does teaching which emphasizes understanding and generalization lead to the more effective learning of arithmetic than instruction which gives these a minor place?

The Studies:

Swenson, Esther J. *Organization and Generalization as Factors in Learning, Transfer and Retroactive Inhibition.*

Anderson, G. Lester. *Quantitative Thinking as Developed under Connectionist and Field Theories of Learning.*

The Source: *University of Minnesota Studies in Education. Number 2, 1949. Learning Theory in School Situations.*

PART I

THE DOMINANT theme in the literature dealing with the teaching and learning of arithmetic today is meaning and understanding. A democratic philosophy and psychology's field theories seem to approve this theme. The two Minnesota studies attempt to put the melody to a classroom test.

Other investigations are related to these two. About 1931 Olander[1] studied the results of two methods of instruction in the hundred addition and subtraction combinations on 1300 beginning second grade children over a period of seventeen weeks. For three minutes per day one group was given instruction in generalizing the zero combinations, using the commutability principle and the inverseness of addition and subtraction. The second group used the three minutes for drill. Both groups used the rest of the class time for drill. The results showed no difference in the abilities of the two groups to transfer their learning to untaught combinations. Olander opined that the function was too narrow to require generalization, that the three minutes of instruction may have been too brief and that perhaps the children were too immature to profit from the special kind of teaching. In the same year Overman[2] reported an investigation of the efficacy of four different methods of instruction, in improving the ability to subtract two-digit numbers and to add and subtract two- and three-digit numbers. The four groups of one hundred and twelve children each had been matched on their ability to add two-digit numbers. The method based upon the development of general procedures produced learning results that surpassed a "show how" method and a rationalization method by statistically significant results. A method based upon a combination of the generalization and rationalization procedures was not as good as generalization alone. Strange as it may seem, the rationalization method, which involved a discussion of underlying principles and reasons for procedures fared no better than the "show how" method. In 1938 Thiele[3] described an experiment involving two groups of about three hundred second grade children of average to low intelligence over the period of a school year. The drill group was taught the hundred basic addition facts in the Clapp order of difficulty by practice on specific facts. The generalization group was taught in a sequence of seven stages, namely, adding one and the reverses, adding two and the reverses, adding zero and the reverses, the doubles, one more than the doubles and one less than the doubles, adding ten and the reverses and adding to nine and the

reverses. Each of these stages was introduced by concrete experiences of the visual-kinesthetic type. Later, practice was given in identifying a combination with a member of the sequence. Each of the two methods was represented by seven teachers. At the end of the school year a test of the hundred addition facts arranged in a mixed order showed that the generalization method was superior in a statistically significant sense to the drill group in both the average and below-average intelligence groups.

The purpose of the Swenson investigation was "to study learning, transfer of training and retroactive inhibition as they appear in the learning of the hundred addition facts by second grade children taught by three different methods of instruction, the chief variable being the degree of emphasis upon organization and generalization in the learning process" (p. 9). Three hundred thirty-two children in fourteen different second grades of St. Paul, Minnesota, were involved over a period of twenty weeks. The classes were assigned at random to the three different teaching methods. Twenty-five minutes of instruction were given daily. In all classes the children worked assigned verbal problems and formulated some of their own. Attention was given to social uses and applications. During the first two weeks instruction in number meanings was given. No subtraction was taught. The investigator met each of the three teacher groups seven times, prepared a teaching manual for each method and visited the teachers to see if the teachers actually followed the assigned methods.

In the generalization method the meaning theory was employed. The teacher encouraged the children to discover and organize arithmetic relationships. Concrete teaching aids were used when needed. Practice was provided during the learning process and after understanding was attained. In the drill method the aim was to learn the specific addition facts. The order of difficulty was based on difficulty studies. Speed of response was a dominant aim. The third method, derived from a study of current textbooks, was a mixture of certain parts of the other two. Concrete aids were used in initial presentations. The manipulation of these and the use of counting were followed by drill. The facts were presented in an order governed by the size of the sum. For instance, all the combinations having five as a sum were considered before any having a sum greater than five.

After the initial two weeks of number readiness instruction a test disclosed that there was no significant difference between the three methods groups. After another week there followed five weeks of instruction on fifty-five to fifty-eight of the hundred number facts. On the last day of this instruction a test revealed that the generalization method was best by a statistically significant amount "for promoting learning during instruction." The drill method was superior to the mixed method. For the next five and one-half weeks twenty-two to twenty-four new facts were taught. The end test revealed no significant difference in gains. During the two and one-half weeks of the Christmas vacation which followed, the retention loss was less for the generalization group than for the other two. During the next four weeks the remaining facts were taught. The test at the end again showed no significant difference in gains but did reveal that the generalization group retained over the seventeen week period a significantly greater amount. Another test given the following day demonstrated that the amount of transfer made by the generalization group to the untaught process of subtraction was significantly greater than that made by the other two groups. Two days later another test involving the untaught processes of carrying, bridging and the adding of one-digit, two-digit and three-digit numbers in the same problem again disclosed the statistically significant advantage of the generalization method for transfer purposes.

This has been only a partial summary of Dr. Swenson's doctoral study. In

addition to supporting the hypothesis that the generalization method leads to greater transfer, less retroactive inhibition and, therefore, more retention, this superb investigation also indicated that the relative difficulty of the addition facts is not fixed but is dependent on the method of learning and that the greater the intelligence the greater the transfer and the less the retroactive inhibition.

Part II

The purpose of the Anderson study was "to investigate the hypothesis that learning that emphasizes understanding and generalization is more efficacious than learning which emphasizes the relative discreteness of the elements of knowledge and skill." (p. 40). . . . "The most important specific problem was to determine the effects of instruction upon the ability to think mathematically in quantitative situations." (p. 40). This is one of the few studies that have attempted to test the emphasis on meaning and understanding beyond the second grade.

Involved in the study were eighteen fourth-grade Minneapolis classes in eighteen schools and eighteen different teachers. Ten classes were taught by a drill method and eight by a meaning method. The experiment ran from November to May. The instruction included the four basic operations and problem solving. The city's syllabus in arithmetic was followed. About four hundred children took part in the investigation.

In the drill classes the subject matter was analyzed into elements. The elements were mastered in isolation and in their final form. Formal repetition was used in the mastering process. In the meaning classes the number system was treated as a system. Emphasis was on the discovery of relations and the formulation of generalizations through experience with the system. The generalizations were tested and fixed through repeated and varied applications.

The teachers were assigned to a particular method if they had practiced it or held a point of view consistent with it. There was no prescription of procedures, content or time distribution since Dr. Anderson desired to make the situation a typical rather than a rigidly controlled one. Ten meetings of ninety minutes each were held with each group of teachers to make the implications of each method clear. The teachers kept logs and prepared written statements at the end. The teachers were also visited to check on the methods used. The minimum age of the teachers was 38 and the minimum experience 17 years.

Test data included an intelligence test and the results of an initial and terminal administration of the Compass Survey Test and the Analytical Scales of Attainment. These provided information on computational growth, problem-solving gain and changes in social concepts and vocabulary of a quantitative nature. There was also a Mathematics Thinking Test with one section dealing with relationships among number facts and another demanding the discovery of relationships in a number series.

The children in each of the two groups were compared on the basis of their initial arithmetic achievement and their intelligence. The final tests revealed changes that were difficult to interpret because the homogeneity of variance necessary for reliable statistical analysis was usually not apparent. In computation the low intelligence drill group attained significant gains over the low intelligence meaning group in addition and subtraction. The high intelligence meaning group made insignificant gains over the high intelligence drill group in addition and subtraction but lost out in multiplication. The low intelligence meaning group made insignificant gains over the low intelligence drill group in subtraction. The results of the analysis of the problem-solving gains were inconclusive. The same conclusion held in the case of the social

(*Continued on page 120*)

NOTES ON THE HISTORY OF MATHEMATICS

Edited by VERA SANFORD
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An Old Problem with a Modern Twist

THE PROBLEM that is the subject of this article has a basis in reality but its conditions are so idealized that it seems unlikely that it could ever have arisen in the form in which it occurs. Yet the reader will find in it a situation that has important applications today.

The problem appears in the *Liber Abaci* (1202) written by Fibonacci:

> A certain trader passed through Lucca and there doubled his money. He then spent 12 denarii. He next passed through Florence where he doubled his money and spent 12 denarii. He then returned to Pisa where he doubled his money and then spent 12 denarii and had nothing left. How much was his original principal?

It should be noted that Fibonacci who is also known as Leonardo of Pisa had his merchant start from Pisa and visit Lucca and Florence neither of which is far away, and return to Pisa again. The use of local names is common in these problems.

Fibonacci had the habit of presenting a problem situation in different forms. He did this in the case of the problem quoted above, referring to the "fairs" which were held annually on specified dates in various centers in Europe. The variants are as follows:

> Again, a man visited three fairs carrying with him $10\frac{1}{2}$ denarii with which, as before, he doubled his money at each fair and he also spent the same amount at each fair and nothing remained. I ask how much he spent at each fair.

> A man who had 13 bezants visited I know not how many fairs, and at each of them he doubled his money and spent 14 bezants. I ask how many fairs he visited? (In this case, the numbers are poorly chosen for he visited $3\frac{3}{4}$ fairs.)

This problem was not restricted to merchants, nor was it used only by Fibonacci. Here are some of the variants:

> A man went into an orchard in which there were seven gates, and there he stole a certain number of apples. When he left the orchard, he gave the first guard half the apples that he had and one apple more. To the second he gave half of his remaining apples and one apple more. He did the same to each of the remaining five guards and left the orchard with one apple. How many apples did he gather in the orchard?—Fibonacci (1202).

> A thief stole a sackful of gulders from a castle. Now the castle had three gates and a gatekeeper stood at each. The thief hurried, anxious to be safely out of the castle with the stolen money. When he came to the first gate, the gatekeeper asked what he carried and said, "Give me half and I'll let you out." The thief in fear lest he be seized, gave the gatekeeper half of the money and the gatekeeper gave him back 100 gulders out of sympathy. At the second gate, the keeper demanded half his money, but returned to him 50 gulders. At the third gate, the keeper demanded half his money and on receiving it gave him back 25 gulders. When the thief got safely out of the castle, he had 100 gulders. The question is how many gulders he first put into his sack?—Köbel (1514).

> A gentleman had an audience with a Signore, and according to custom, he gave each of the guards one tenth of the money he had with him. When he came, he had 100 florins. If he gave one tenth of his money to each of ten guards, how much did he have left when he departed? (The result is given as $34\frac{86784401}{100000000}$ florins.)—Ghaligai (1521).

> A man who had a certain capital fell to gambling and made as much money as he had to start with. He then spent 20 ducats on a horse. He rode away on the horse to an inn where he gambled with the inn-keeper and redoubled his money. He spent 20 ducats on a beautiful robe. He then left the inn and went on to the gate of the city where he found some people gambling. There he doubled what he had left, bought a ring for 20 ducats, and found he had nothing left. How much money did he have when he started from home?—Tartaglia (1556).

Tartaglia gives the situation a different application in this example:

A merchant gives a university 2814 ducats on the understanding that he will pay 618 ducats a year for nine years at the end of which time the 2814 ducats will be considered as paid. What interest is the merchant getting on his money? (The rate is over 19%.)

It will be noticed that while the other instances can be handled by simple computation, this last illustration is much more complicated. Yet it is a logical extension of the other situation with an unknown rate of gain being substituted for the doubling of the other cases.

The common element in these problems is that starting with a certain sum of money, this is increased (or decreased) at a fixed rate (generally doubled or halved), a fixed sum is paid out, and this process is repeated a certain number of times and a certain amount (or zero) is left.

Parenthetically, it should be added that Fibonacci had still another twist to give the problem:

A man who was approaching his end, called his eldest son to him and said "Divide my estate among yourselves in this way: You are to have one bezant and a seventh of the rest of my property." He said to the second son "You are to have two bezants and one seventh of what then remains." To the next one he gave three bezants and a seventh of what was left. And thus he called his sons in order and gave to each, one bezant more then the one before and a seventh of his property beside, and the last one had all that was left. It happened, moreover, that each had an equal share in the estate according to these conditions. The question is how many sons were there and how large was the estate?

The emphasis these questions received can be explained by the interest their apparent paradoxes would have: a man has nothing left after spending constant amounts from capital that is always doubling; he can give away one tenth of his money ten times and have something left; he can adopt the peculiar terms of the increasing number of bezants to each son and a seventh part in a dwindling estate and yet have the sons actually share equally. Thus far they are puzzles. But when put in terms of a sum of money which draws a fixed rate of interest and out of which a fixed sum is to be paid each year, we have the idea of an annuity. How much money must be set aside to guarantee the payment of a certain amount for a specified number of years, the interest rate being given? For how many years can such and such a sum be collected annually if a certain sum is invested at a given rate? What interest rate must be earned, if a fixed sum is to yield so and so much money annually for a specified number of years? It is likely that few other problems show evolution from a group that once contained only puzzles of varying degrees of reality, to a group that now has important genuine problems of great importance.

Research

(*Continued from page 118*)

concepts and vocabulary having quantitative connotations. In the Mathematics Thinking Test the significant differences seemed in favor of the drill group scoring low in intelligence but above average in the arithmetic pretest and to the advantage of the meaning group above the average in intelligence but below the mean in the arithmetic pre-test.

In summarizing, Dr. Anderson asks whether his study means that a meaning theory of instruction is better for the group above the average in intelligence but below in arithmetic achievement and whether the kind of teaching which emphasizes drill procedures is better for the group which is low in intelligence but above average in arithmetic achievement.

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MATHEMATICAL MISCELLANEA

Edited by Phillip S. Jones
University of Michigan, Ann Arbor, Michigan

43. A New Proof of an Old Theorem

Although many proofs of the following theorem have been written, the author believes that the indirect proof given here is both interesting and unique.[1]

Theorem: *A necessary and sufficient condition that a triangle be isosceles is that the bisectors of the base angles be equal.*

Proof: In any triangle, the bisector of an interior angle divides the opposite side internally into segments proportional to the adjacent sides of the triangle.

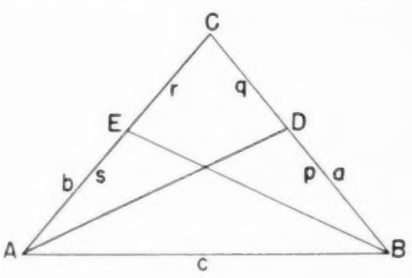

Fig. 1

In the triangle of Figure 1 this gives us, assuming AD and BE to be bisectors of angles A and B respectively:

$$\frac{q}{p}=\frac{b}{c} \quad \text{or} \quad qc=pb. \tag{1}$$

Substituting $p=a-q$ in (1) we have

$$q=\frac{ab}{b+c}\,. \tag{2a}$$

[1] *Department editor's note:* For example, proofs may be found on pages 141 and 142 of *An Introduction to Modern Geometry* by Levi S. Shively (New York: John Wiley and Sons, Inc., 1939). The second proof in this book is based on Stewart's Theorem and has much in common with Mr. Sevier's given here, though it is not the same.

Similarly one can easily derive

$$r=\frac{ab}{a+c} \tag{2b}$$

$$s=\frac{bc}{a+c} \tag{2c}$$

$$p=\frac{ac}{b+c}\,. \tag{2d}$$

Now suppose we circumscribe a circle about the given triangle and draw the bisector of the remaining angle (Fig. 2). Let this bisector meet the opposite side in F and the circumcircle in G.

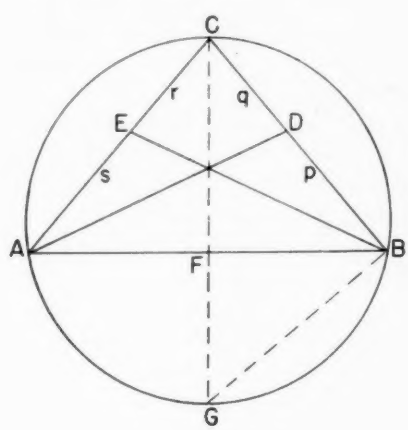

Fig. 2

If we join B and G we have that $\Delta AFC \sim \Delta GBC$ from which we obtain:

$$\frac{\overline{CF}}{a}=\frac{b}{\overline{CG}} \quad \text{or} \quad \overline{CF}\cdot\overline{CG}=ab. \tag{3}$$

Since $\overline{CG}=\overline{CF}+\overline{FG}$, by substituting in (3) and solving for $\overline{CF}^2$ we have

$$\overline{CF}^2=ab-\overline{CF}\cdot\overline{FG}. \tag{4}$$

That $\overline{CF}\cdot\overline{FG}=\overline{AF}\cdot\overline{FB}$ may also be seen from Figure 2. Substituting in (4) we have

(5a) $\overline{CF}^2=ab-\overline{AF}\cdot\overline{FB}.$

The following could be deduced in a similar fashion

(5b) $\overline{AD}^2=bc-pq$

(5c) $\overline{BE}^2=ac-rs.$

Now since $\overline{AD}=\overline{BE}$ by our original hypothesis, $\overline{AD}^2=\overline{BE}^2$, and equating (5b) and (5c) gives

(6) $bc-pq=ac-rs$

which may be written as

(7) $c(b-a)=pq-rs.$

Substituting (2a), (2b), (2c), (2d), in (7) and simplifying gives

(8) $$c(b-a)=\frac{a^2bc(a+c)^2-ab^2c(b+c)^2}{(a+c)^2\cdot(b+c)^2}\,.$$

If now $b>a$ the left member of (8) is positive, and the right member is negative, a contradiction. A similar contradiction arises if $b<a$.

Hence, $b=a$, and the triangle is isosceles. In this case (8) is satisfied, both members being zero.

Thus the sufficiency of the condition of our theorem is proved. The necessity is well known and the proof is omitted.

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44. Reduction Formulas

Two methods are commonly used in introducing the study of trigonometric functions. The method used in most of the older texts is to begin with the acute angle and define the functions in terms of the sides of a right triangle containing this acute angle. A later method which has become increasingly more popular, especially in college courses, is to begin with the general angle connected with a rectangular coordinate system, and to define the functions from the start in terms of x, y, and r.

Whichever method is used, however, the student must learn fairly early that for every angle greater than 90°, except a quadrantal angle, there is an acute angle whose functions have the same numerical values as those of the given angle, and that, for this reason, tables of trigonometric functions need go only from 0° to 90°.

To enable the student to use these tables for angles of any size most texts give "reduction formulas," sometimes to the exclusion of other methods. In every text that I have seen, there are usually about 15 to 20 pages given to this subject. The student enters a maze of *90−θ's, 90+θ's, 180−θ's, 180+θ's,* $n\cdot 90\pm\theta$*'s* and accompanying theorems. When he finishes this work he is supposed to be able to find the trigonometric values of any angle from tables that go only to 90°.

Many of the students, however, never seem to get through this maze without a lot of wasted time. To remedy this, I have tried a different method and have had much success using it.

Consider the angle being studied to be referred to a rectangular Cartesian coordinate system with its vertex at the origin and its initial side along the positive x axis as in Figure 3.

I then define the acute angle formed by

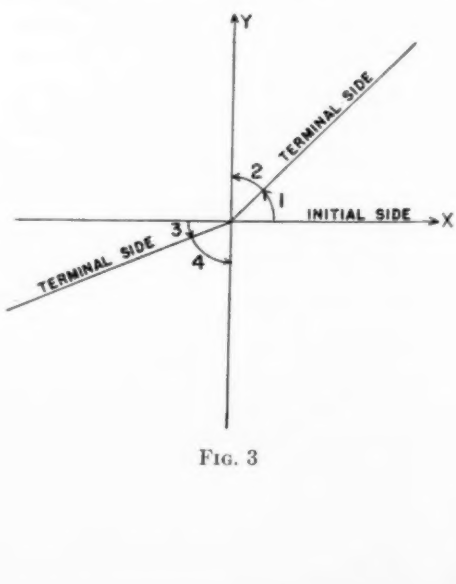

Fig. 3

the x axis and the terminal side to be *angle-sub-X*[2] and the acute angle formed by the Y axis and the terminal side to be *angle-sub-Y* which I represent by $\angle_x$ and $\angle_y$ respectively. Angles 1 and 3 in Figure 3 are examples of $\angle_x$ and angles 2 and 4 are examples of $\angle_y$.

Consider any angle as A in Figure 4, with terminal side OF. From any point P_1 with coordinates (X_1, Y_1) on OF, construct P_1D perpendicular to the X axis. Angle DOF is then $\angle_x$.

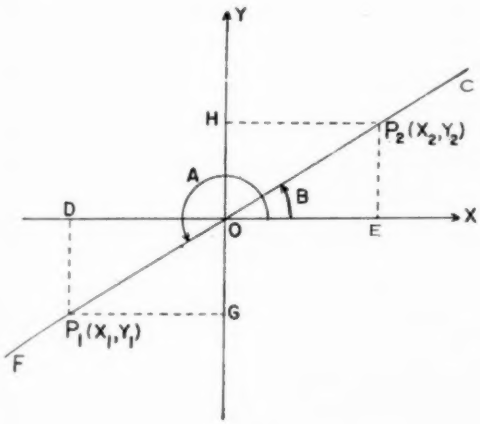

FIG. 4

Construct angle B in the first quadrant with terminal side OC, equal to $\angle_x$. With $OP_2=OP_1$, construct P_2E perpendicular to the X axis. Then $\Delta DOP_1 \cong \Delta EOP_2$.

Hence, the coordinates of $P_1(X_1, Y_1)$ are numerically equal to the coordinates of $P_2(X_2, Y_2)$.

Construct the lines P_1G and P_2H perpendicular to the Y axis. Angle GOF is then $\angle_y$ and is equal to angle HOC since both are complements of equal angles.

Since ODP_1G and OEP_2H are rectangles it is easily seen that the trigonometric functions of $\angle_x$ are equal to the co-functions of $\angle_y$.

Hence we have the theorem: *The trigonometric functions of any angle are numerically equal to the same trigonometric functions of $\angle_x$ or the co-functions of $\angle_y$.*

The sign of the original trigonometric function can also be easily seen by recalling the basic definitions of the function and considering the signs of the coordinates X_1 and Y_1. If we define $OP_1=r=\sqrt{X_1^2+Y_1^2}$, then Sin $A=Y_1/r$, Cos $A=X_1/r$, Tan $A=Y_1/X_1$, etc. The signs of X_1 and Y_1 determine the signs of the functions in accordance with the position of the terminal line and the sign laws of algebra. The absolute values of the functions are determined by the following relations, reasoned, not memorized:

$$|\text{Sin } A| = \text{Sin } \angle_x = \text{Cos } \angle_y$$
$$|\text{Cos } A| = \text{Cos } \angle_x = \text{Sin } \angle_y$$
$$|\text{Tan } A| = \text{Tan } \angle_x = \text{Cot } \angle_y$$
etc.

Using this new theorem it is easier to teach that the functions of any angle can be found in terms of the functions of a positive angle less than or equal to 45° (either $\angle_x$ or $\angle_y$ must be less than or equal to 45°). Moreover, students grasp the proof and use it with less difficulty because it reduces the memory work required and eliminates the difficulties involved in the selection of the proper reduction formulas.

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45. Selecting and Using Formulas for Composite Areas

Many formulas for finding the areas of plane geometric figures are taught in junior high school as a series of isolated topics. In a review test, it is desirable to see if students have grasped the main idea behind each formula by presenting them with data and problems such as the following.

Formulas to use:

(*Continued on page 129*)

[2] *Department editor's note:* The term *reference angle* is used in many college texts for Mr. Gorsline's *angle-sub-X* and hence may be a little more desirable terminology.

AIDS TO TEACHING

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CHARTS

C. 29—Adding Signed Numbers

C. 30—Short Cuts in Problem Solving

Scott, Foresman and Company, Chicago 11, Illinois

Classroom charts; Free

Description of C. 29: This chart ($17\frac{1}{2}''\times 21''$) contains two scales, each $2\frac{1}{4}''\times 15\frac{1}{2}''$, which can be mounted on cardboard and manipulated to add signed numbers. Each scale runs from -9 to 9. A useful text containing teaching suggestions is included in the top part of the chart.

Appraisal of C. 29: It is unfortunate that a larger set of scales could not have been provided so that the figures could be large enough to be seen from the back of a classroom and so that the number scales could extend to larger numbers. In order to do this the suggestions to teachers could be printed in a separate pamphlet to accompany the chart. Six inches by 48 inches would have been more appropriate. However, the idea is stimulating and the construction of larger scales by teacher and pupils would not be difficult.

Description of C. 30: This chart ($14''\times 20''$) shows a series of six cartoons of a boy and a girl with their teacher doing some arithmetic problems. In order to shorten the work it is suggested that two computational steps be combined into one when possible; that cancellation be used; and that numbers be rounded off before they be used in computation.

Appraisal of C. 30: The cartoon technique is eyecatching; the message is interesting. Some teapot tempests will be stirred up by the use of "cancellation," but such purists need not use the chart.

C. 31—Dividing Factors for Square Root

C. 32—Multiplying Factors for Square Root

Monroe Calculating Machine Company, Orange, New Jersey

Cardboard tables; $7''\times 11''$; Free

Description of C. 31: By using only the first three digits from a number, a factor is determined with seven digits which will find the square root of the original number to five significant figure accuracy with one performed division.

Description of C. 32: This table is similar to C. 31, only a factor is given which supplies the square root by multiplying.

Appraisal of C. 31 and C. 32: The ingenuity of devising a method which reduces square root to an operation of dividing or multiplying is to be admired, but these tables should not be sent for unless the school owns a machine where they can be put into practice. They have little meaning without this opportunity.

C. 33—How Life Insurance Meets Family Needs

C. 34—Life Insurance Dollars at Work

C. 35—How America's Families Use Life Insurance

Institute of Life Insurance, 488 Madison Avenue, New York 22, N. Y.

Charts; $27''\times 38''$; \$0.15 each

Description of C. 33: On the background of a huge slate a six color poster shows how term insurance, straight life, limited payment life, and endowment differ with respect to protection for dependents and for the insured during his lifetime. The bene-

fits are figured for $100 annual premium at age 25.

Description of C. 34: This blue and yellow chart shows how the assets of an insurance company are invested in business and industry, government, real estate and miscellaneous projects.

Description of C. 35: The uses of life insurance are outlined in this chart.

Appraisal of C. 33, C. 34, and C. 35: It must be admitted that these charts are some of the most attractive and effective that are now available. Likewise, it must be admitted that they are pure propaganda for buying life insurance; this is fair enough since they are financed by life insurance companies. But, in teaching one must constantly stress the other ways that one can provide the same benefits, the other types of investments. They would be better charts for educational purposes if this could be incorporated.

Specifically, C. 33 is a little difficult to read with its particular color combinations, but is graphically dramatic. C. 34 shows the percentage of funds in each category in a very effective manner, but it is not a subject that is very important to teach in school. C. 35 is rather poorly organized and repetitious, but brings together many interesting reasons for insurance to lead to discussion.

FILM

F. 67—The Impossible Map

Producer: National Film Board of Canada, 400 West Madison St., Chicago 6, Ill.

Educational Collaborator: Miss Evelyn Lambert

B&W ($30.00) and color ($75.00); 1 reel; 10 min.

Description: The opening scene shows a globe slowly turning around. A grapefruit, paint and brushes, a knife, transparent celluloid sheet and paper are used to show how difficult it is to make an accurate representation of the world on a flat surface.

A young man is trying to flatten out a grapefruit with a rolling pin, but does not seem too successful. The grapefruit, painted to approximate a globe, is sliced and peeled. He lets the strips fall on the floor. There are many gaps. The pieces are arranged under the celluloid and the painted area copied. The result is accurate in the center, but distorted toward the poles. The strips are arranged in another way and the result is a good picture of the Americas. The rest of the world is distorted. The strips are then cut into sixteenths. Drawings are made of South and North America, Europe, Africa and Asia, China, and New Guinea and Australia on the celluloid. The celluloid is then flattened out and the well-known mercator map is shown. The pieces are then rearranged around the North Pole. They are flattened out again and there is a good picture of the North Pole.

With these various areas tried it is obvious that there are many difficulties of representing the globe's land areas on a flat surface. The globe is the only correct way to show the whole world.

Appraisal: Twelve teachers of mathematics were present at the evaluation. Considering the entire film 50% or more found that it was excellent for introducing new material, to augment explanations and to motivate, but that it was only fair for developing skills. 85% of the teachers thought the film was suited for grades 7 through 11, however, 44% of these favored grades 7 and 8. Algebra, Arithmetic and Trigonometry were the courses chosen as most favorable for this film. 90% of the teachers found no inaccuracies in the film. Two-thirds of the teachers thought the film would completely hold the attention of the students while the remaining one-third thought it would partially hold their attention. It was interesting to note that 92% of the teachers thought that the content of the film could not be just as effectively and efficiently presented in some other way. Three-fourths of the teachers said they would use this film in their classroom. (Reviewed by A. DiLuna, R. Fleet, K. Hathaway)

FILMSTRIPS

FS. 50A—Multiplication and Division of Signed Numbers

Society for Visual Education, 1345 Diversey Pkwy., Chicago 14, Ill.

B&W ($3.00); 36 frames.

Description: The first section of the filmstrip points out that multiplication of a positive number by another positive number is continued addition. It then indicates that when signed numbers are multiplied one of the following combinations of signs will occur: $(+)(+)$; $(-)(+)$; $(+)(-)$; $(-)(-)$. From the usual definition of multiplication in arithmetic the filmstrip demonstrates that $(-2)(+6)$ gives -12, and generalizes to $(-)(+)$ gives $(-)$. It then poses the problem of a positive quantity used a negative number of times. This is followed by the statement that in mathematics many expressions have no meaning until we *agree* upon some meaning for them. Using pictures in the frames, these agreements are then set forth: law abiding citizens are *positive;* law breakers are *negative;* being rewarded is considered *positive;* being punished is considered *negative*. These agreements are combined to get,

"If good citizens are punished,
(+) (−)
that's bad."
= (−)

"In other words," says this filmstrip, "If we multiply a positive number by a negative one, the result is a negative number." It goes on to say,

"If law breakers are punished,
(−) (−)
that's good."
= (+)

From this it is asserted that the product of two negative numbers is a positive number. The four rules of signs for multiplication are then combined to the usual two rules. The laws of division are treated thus:

"Punishment divided among
(−) ÷
law breakers is good."
(−) = (+)

"A reward divided among
(+) ÷
law abiding citizens is good."
(+) = (+)

"Punishment divided among
(−) ÷
good citizens is bad."
(+) = (−)

"A reward divided among
(+) ÷
law breakers is bad."
(−) = (−)

These give, "The quotient of two numbers which have like signs is positive and the quotient of two numbers which have unlike signs is negative." At the end of the filmstrip a brief practice on using these rules is provided.

Appraisal: From the above description of content it is obviously unnecessary for the reviewer to give a detailed evaluation. Teachers who desire that their elementary algebra pupils receive a firm foundation for future work in mathematics will recognize the danger in the "explanation" given by this filmstrip. (Reviewed by Frances Burns, Oneida High School, Oneida, N. Y.)

FS. 104—Cubic Measure—Volume

Photo and Sound Productions, 116 Natoma Street, San Francisco 5, Cal.

Educational collaborator: O. W. McGuire.

B&W ($4.00); 32 frames.

Description: This is Part VII of the Study of Measurement series. In the opening frames a box and a book are pictured as illustrations of solids. The vocabulary of length, width, thickness and dimension is introduced with respect to a rectangular

solid. In succeeding frames, face and base of a solid are given; rectangular solid is defined; area of the solid is explained. The distinction between base (line) of a plane figure and base (area) of a solid is pointed out. The cubic inch, the cubic foot and the cubic yard as units of cubic measure are pictured in separate frames and then together in one frame for comparison. Volume of a solid is defined as the amount of space it occupies as measured in cubic units. Cylinder and sphere are also pictured as solids. On a rectangle whose area is twelve square inches, a solid is built by using one layer of cubic inch units, then two layers, then three layers thus obtaining volumes of twelve, twenty-four, and thirty-six cubic inches. It is now stated that the volume of the solid is equal to the area of the base times the height. This is written in symbols as $V=Bh$. The formula is now used to find the volume of a cylinder. The derivation of 1728 cubic inches in one cubic foot and of twenty-seven cubic feet in one cubic yard is pictured. A cubic yard of earth as a "load" is shown as an application of cubic measure. The final frame repeats the formulas $V=lwh$ and $V=Bh$.

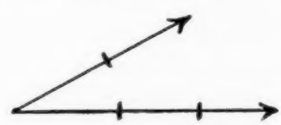
Appraisal: The material covered and the treatment are suitable for intermediate and junior high school pupils. With the following two exceptions it is well presented. One is the assumption that the volume method as developed for a rectangular solid will hold also for a cylinder. The other is the use of $\pi=3.1416$ with a circle whose radius is three units. Pupils would get much more complete and lasting understanding from actual manipulation of the objects than from a study of the filmstrip. It could, however, be used to advantage in classes where such tactual experience is not provided. It could give beginning teachers, without special training in mathematics, suggestions for collecting materials and for a method of presenting the topic. (Reviewed by Frances Burns, Oneida High School, Oneida, N. Y.)

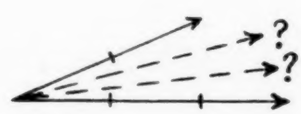
FS. 105—Vectors

The Jam Handy Organization, 2821 East Grand Boulevard, Detroit 11, Mich.

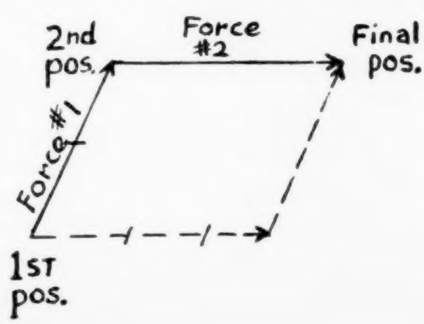
B&W ($4.00); 55 frames.

Description: Frames 1–20 answer the question, "What are vectors?" After a brief summary of the properties of parallelograms, we see two applications of the parallelogram—how a pilot uses a parallelogram to find track, given compass course and wind direction; and how an engineer finds the result of several forces acting on an object. Following this introduction, a detailed description of two forces acting on a body begins. One force is a propeller pulling an airplane, and the other is the wind. To simplify the problem, the propeller's force is given as 3 m.p.h. and the wind's as 2 m.p.h. A scale is used to show the length of these directions.

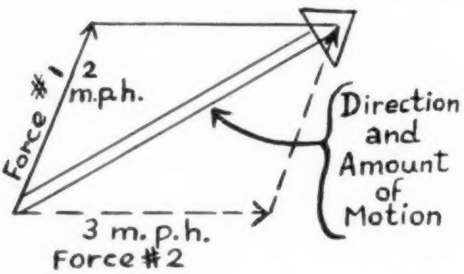


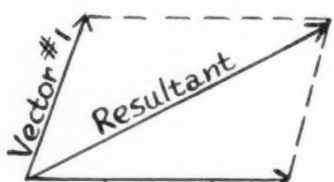


Where will the object move? If one force acted at a time, how can we find the final position? If force No. 2 acted after the first, the direction would be the same, that is, parallel to it. When we draw a line from the final position to the end of force No. 2 we have a parallelogram. So, with the



forces acting at the same time, the object will have moved at the angle of the diagonal and a distance equal to this diagonal. In our parallelogram of forces





and motion we call each force a VECTOR and the diagonal a RESULTANT. The original position is called the POINT OF APPLICATION. Frames 21–40 are devoted to vector uses. The four uses mentioned are as follows:

I. Given the direction of one force or the resultant from a reference line on a map (compass direction), and the amount of two forces and the resultant, we can construct a triangle with the use of dividers. From this we complete the parallelogram, on which we can measure anything else we want to know. We can measure angles with a protractor to find: (1) direction of resultant, (2) direction of second force, or (3) the angle between the forces. (With two known forces and the resultant, we must draw the forces in the right order or we will get the wrong parallelogram.)

II. If we are given two forces and the included angle we can draw the parallelogram and resultant. Measure the resultant with a ruler and the direction by measuring the angle with a protractor.

III. To illustrate given one force, the resultant and angle between them, an automobile is shown on an inclined plane. The resultant is acceleration and force No. 1 is gravity. The triangle is completed first, then the parallelogram. From this, the other vector and its direction can be measured.

IV. In the final illustration we are given the resultant and the angles it forms with the two vectors. Here, the resultant is an airplane's ground speed, and the angles are heading and wind direction. We can measure the two vectors, wind velocity and air speed, from the completed parallelogram.

Frames 41–44 deal with three or more forces. How can we construct a single resultant from three vectors? Find a partial resultant from two vectors and use the partial resultant and remaining vector to form a new parallelogram. The diagonal of the last parallelogram is the resultant. Frames 45–50 present vector addition. Given the direction and amount of each force find the resultant by addition. This saves the time of drawing complete parallelograms. Add vectors by drawing one after another, as long as each keeps its original direction. The line from the original to the final position is the resultant. Frames 51–55 are concerned with computation by trigonometry. Scaling the parallelogram is only as accurate as the drawing and measuring. For absolute accuracy, the engineer must use trigonometry to measure the forces and angles. The law of sines will give him the necessary formula for all vector problems when the forces and their directions are known.

Appraisal: This is an interesting and instructional filmstrip, but requires additional explanations and instruction by the teacher. The filmstrip tells a lot in a short time, and some of the facts are too deep for immediate consumption. The applications are very good, but more class time was needed to complete the discussion. Pupils who had studied physics found the filmstrip extremely interesting. Most of the pupils enjoyed the applications. The filmstrip developed a greater appreciation

for the parallelogram and its applications. Some pupils expressed the opinion that a clearer picture would result if the illustration were printed as white on black rather than black on white. Some teachers felt that the term vector should be defined as well as illustrated. Further illustrations of trigonometric solutions, including the law of tangents, would add to the appeal of this filmstrip to the students. (Reviewed by Herbert Freed, Senior High School, Atlantic City, N. J. Additional comments by Mabel V. Rhodes, Senior High School, Atlantic City, N. J.)

Mathematical Miscellanea

(Continued from page 123)

1. $A = l_w$
2. $A = \pi r^2$
3. $A = \frac{1}{2}bh$
4. $A = s^2$
5. $A = \frac{1}{2}h(b_1+b_2)$
6. $\frac{A}{2} = \frac{\pi r^2}{2}$.

Problem I. Find the area of the cross-section of the ice cream cone shown in Figure 5.

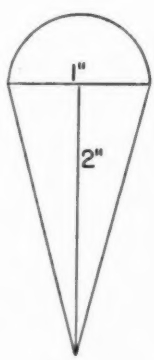


FIG. 5

Problem II. Find the area of the cross-section of a sailing boat shown in Figure 6.

The value of such work lies in its interest arousing quality plus the requirement that the student select the proper formulas and determine dimensions from the diagram.

For superior or older students the diagram may be labeled, as blue prints often are, such that some dimensions needed are given only implicitly. Also algebra students can make up literal formulas for a composite figure before substituting numerical values.

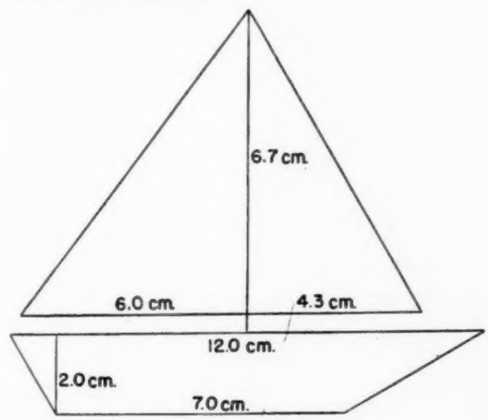


FIG. 6

Students can also be shown examples of composite areas on blue prints and can be told of how cross-section areas computed at a number of heights are used to determine volumes and thence centers of gravity of ships. They can be told how the volume, V, of the ice cream cone of Figure 5 can be computed as a volume of revolution if we know the area, A, and center of gravity of one of its halves ($V = 2\pi rA$, where r is the distance from the axis of rotation to the center of gravity). Such problems, then, can be adapted to the interests and abilities of many different students of varied abilities and at different grade levels.

GEORGE JANICKI
Elm School
Elmwood Park, Illinois

The Thirtieth Annual Meeting of the N.C.T.M.

Be sure to study the program on pages 145–156 of this issue.
Show it to your superintendent and principal.
Discuss it with other teachers of mathematics in elementary and secondary schools.
Will your school have at least one representative at Des Moines?

WHAT IS GOING ON IN YOUR SCHOOL?

Edited by John R. Mayor *and* John A. Brown
The University of Wisconsin, Madison, Wisconsin

Preliminary Report on Replies to Questions on General Mathematics and Third Year Mathematics

Sets of questions on Mathematics Enrollments, General Mathematics, and Third Year Mathematics were published in this Department during the school year 1950–51 and were repeated in the November, 1951 number. A final summary of answers will be published in the May number. Reports on early replies to the questions on Mathematics Enrollments have been given in the April and November, 1951, issues. A preliminary report on early replies to the questions on General Mathematics and Third Year Mathematics will be found in the following paragraphs, even though the number of replies is too small to provide a basis for general conclusions.

Nearly eighty per cent of the first thirty-two replies on General Mathematics indicate that the schools for which the reports were made offer both algebra and general mathematics in the ninth grade. For these schools replying to the question on what part of their students take general mathematics as their first course in high school mathematics, the per cents range from eight per cent to one hundred per cent. More than half of those replying indicate that less than half of their students take general mathematics as the first course in high school mathematics. The replies show a great difference in methods used for placement of students in general mathematics. Eight of those replying state that their students may take a second year of general mathematics.

In the first twenty replies to the questions on Third Year Mathematics the schools are almost equally divided among those in which the third year course is a third semester of algebra and a semester of solid geometry, a third semester of algebra and a semester of trigonometry, a second year of algebra, and a year of plane geometry. No school reported ability grouping in third year mathematics. Four schools among those reporting have mathematics clubs. Replies show considerable variation in the use of films in third year mathematics, which may of course be determined in part by the courses offered. Eight of those replying state that their students do not use the library as a part of the suggested study in the course while five of those replying indicate some use of the library by their students. Seven did not answer this question.

Science Talent Search

Announcements for the Eleventh Science Talent Search sponsored by the Westinghouse Electric Corporation and Science Service indicate that in quite a number of schools mathematics students took part in the Tenth Science Talent Search, for the school year 1950–51, and were successful in the contest. While more science students than mathematics students enter the competition there is always a good representation of students, both in quantity and quality, who take part because of the encouragement of mathematics teachers and who write re-

ports on their scientific projects, choosing mathematics topics.

Among the titles of the reports of the forty boys and girls who won trips to Washington in last year's Science Talent Search are the following of special interest to mathematics teachers:

A Graphical Development of the Sieve of Eratosthenes—Paul Maxim Braverman, 17, Brooklyn Technical H. S., Brooklyn, N.Y.

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Effects of Absorption and Geometry on the Beta Count Rate—Robert Alan Spangler, 17, Celina H. S., Celina, Ohio.

A further indication of the past participation of mathematics students in our schools in the Science Talent Search is given by the distribution of careers of the four hundred young men and women who have been trip winners in the first ten Science Talent Searches. Records show that twenty-two men and three women among the winners have chosen careers in mathematics and, of course, many more have chosen careers in areas such as astronomy, chemistry, engineering and physics in which extended mathematical training has been necessary. Though all winners are even now less than twenty-eight years of age and many had war-time interruptions, twenty-three have already earned M.D., Ph.D., or D.Sc. degrees.

In 1951, Science Talent Searches were held in twenty-one states concurrently with the national competition by special arrangement with Science Clubs of America. This cooperation greatly increases the opportunity for encouragement and recognition of science and mathematics achievement for the boys and girls in our secondary schools. Teachers of mathematics should encourage promising students to take part in the state and national competitions.

Mathematics at Central High School, Omaha, Nebraska

In the Spring an arithmetic test is given to all eighth grade students in Omaha. On the basis of the results from that test, recommendations of the eighth grade teachers and the will of the parents, students are enrolled in general mathematics or algebra in the high school in their district.

At Central High School one year of general mathematics is offered on the freshman level (9th grade). During the first month of the semester there are some shifts between these classes and those in algebra, and a few of the general mathematics students go into algebra in their sophomore year.

Central High School also maintains the four year program of algebra in the ninth grade, plane geometry in the tenth grade, a second year of algebra in the eleventh grade, and semester courses of solid geometry and trigonometry in the twelfth grade. Also in the twelfth year a semester of refresher arithmetic is compulsory for those who do not pass an arithmetic test given in December.

Two of the most interesting innovations were (1) the administration of final examinations in mathematics three weeks before the end of the semester so that the remaining weeks could be spent on remedial teaching, and (2) the exhibit which was held along with the annual science exhibit

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and open house. The experiment with early examinations proved quite successful and probably will be continued. The exhibit offered excellent opportunities for displaying the extensive mathematical background for science and also the use of mathematics in high school physics and chemistry. The first exhibit was worked up on the spur of the moment and offered only passing interest in comparison with the more active displays in the science laboratories, but, with more time, it may be possible to work up a more attractive exhibit.

The school provides a transit and two sextants which are used in the plane geometry and trigonometry classes. Bulletin boards are used in two of the rooms to display articles on current mathematical and scientific developments, uses of mathematics, puzzle problems and fallacies, commercially prepared and student prepared drawings.

About one-fourth of the out-of-class time in solid geometry is given to the preparation of a project the topic for which is chosen by the student. It may be a paper tracing the historical development of geometry, the construction of a group of related models, a modified study of non-Euclidean geometry, or some other topic of interest to the student.

Reported by Virginia Lee Pratt
Central High School, Omaha, Nebraska

Special Classes and a Dual Mathematics Club in Shortridge High School, Indianapolis

The Technical Class.—This class was organized in January, 1950, for freshman boys whose vocational aspirations were of a technical nature, i.e. in such fields as engineering and mining. Since its initiation the class has had very little change in personnel. A member of the Physics Department is the teacher of the class, and most lessons are accompanied with physical equipment to illustrate principles of mathematics. Angles, arcs, and circles are studied and illustrated with the use of lenses and mirrors; the Pythagorean Theorem is studied and used in connection with stress, strain, and the coefficient of expansion of wire; the formula, $S=\frac{1}{2}gt^2$ and others, are illustrated with "home-made" devices. The class has about thirty members, and the plan is to have the group study all high school mathematics from the technical point of view. The course will terminate in 1953 and is an attempt at integrating mathematics with science.

Algebra IV (*Special*).—This course is designed, especially, for those students who wish to take the College Board Examinations. The first six weeks' work consists of a topical review of plane geometry and algebra, through quadratics, with a daily test. An American Council of Education Co-operative Test is given at the end of the review of each subject. This activity "acclimates" the students to taking tests (or makes him "test-wise"). For the last twelve weeks of the course, only the pupils who are enrolled for credit remain. During this latter period the usual topics, such as complex numbers, logarithms, series, and the binomial theorem are studied.

Dual Mathematics Club.—Any student is eligible for membership in the Shortridge Hi-Pi Club. This club is generally composed of freshmen, and its meetings are planned to appeal to the average underclassman; however, membership in the regular Mathematics Club is restrictive. For membership in the regular club, the following conditions must be fulfilled:

1. Student must be enrolled in Geometry II, or above, and must have a B average in mathematics;
2. Student must have been a member of the Hi-Pi Club for at least one year, or he must write a research paper on some phase of mathematics, or related field, and submit this paper as a request for membership.

The regular Mathematics Club has been in continuous existence since 1918.

(*Continued on page 136*)

BOOK SECTION

Edited by JOSEPH STIPANOWICH
Western Illinois State College, Macomb, Illinois

BOOKS RECEIVED

College

ALGEBRA

Intermediate Algebra for Colleges, by Joseph B. Rosenbach and Edwin A. Whitman, both of Carnegie Institute of Technology. Cloth, x+219 pages, answers, 1951. Ginn and Company, Statler Building, Boston 17, Mass. $3.00.

CALCULUS

Calculus (Rev. ed.), by Joseph V. McKelvey, Iowa State College. Cloth, vii+405 pages, 1951. Macmillan Company, 60 Fifth Avenue, New York 11, N. Y. $4.50.

MATHEMATICS OF INVESTMENT

Mathematics of Investment, by Paul R. Rider, Washington University; and Carl H. Fischer, University of Michigan. Cloth, xi+359 pages, 1951. Rinehart and Company, Inc., 232 Madison Avenue, New York 16, N. Y. $5.00.

Mathematics of Finance (3rd ed.), by Thomas M. Simpson, University of Florida; Zareh M. Pirenian, University of Florida; and Bolling H. Crenshaw, Alabama Polytechnic Institute. Cloth, xv+335 pages, tables, 1951. Prentice-Hall, Inc., 70 Fifth Avenue, New York 11, N. Y. $4.75.

ADVANCED MATHEMATICS

Advanced Engineering Mathematics, by C. R. Wylie, Jr., University of Utah. Cloth, xiii+640 pages, 1951. McGraw-Hill Book Company, 330 West 42nd Street, New York 18, N. Y. $7.50.

Symbolic Logic, by Clarence I. Lewis, Harvard University; and Cooper H. Langford, University of Michigan. Cloth, viii+504 pages, 1951 (reissue of 1932 printing). Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $4.50.

Tensor Analysis—Theory and Applications, by I. S. Sokolnikoff, University of California, Los Angeles. Cloth, ix+335 pages, 1951. John Wiley and Sons, 440 Fourth Avenue, New York 16, N. Y. $6.00.

Miscellaneous

Offerings and Enrollments in High-School Subjects (Chapter 5, 1948–49, of the Biennial Survey of Education in the United States, 1948–50), by Mabel C. Rice, Robert C. Story, J. Dan Hull, and Grace S. Wright. Paper, vi+118 pages, 1951. Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C. $0.30.

Space—Time—Matter, by Hermann Weyl, translated from the German by Henry L. Brose. Cloth, xviii+330 pages, 1950 (reissue of 1922 printing). Dover Publications, Inc., 1780 Broadway, New York 19, N. Y. $3.95.

REVIEWS

Row-Peterson Arithmetic, Primer, Harry G. Wheat, Geraldine Kauffman, and Harl R. Douglass. Evanston: Row, Peterson and Company, 1951. With Workbook, paper. 64 pp., Primer, $0.60; Workbook, $0.40.

The outside covers of these books are so attractive they invite investigation. On the outside cover of the Primer are gaily colored pictures illustrating the concepts of "up and down," "over and under," etc. Throughout the book the pictures are attractive, interesting, and show action.

The Primer is very good for teaching number readiness and also helpful in reading readiness. It is simple and easy, will teach children to follow directions. It teaches counting, comparing groups apart and putting together. It teaches long, short, more, less, few, etc., and how to count money so that children can understand. It teaches ordinals in an easy manner. There are suggested supplementary activities which are very good to follow up. It covers all the arithmetic children should learn in the Low First.

The Workbook Primer is a perfect parallel for the Arithmetic Primer.—MRS. LORENA HOLDER, John H. Reagan School, Dallas, Texas.

Row-Peterson Arithmetic, Book One, Harry G. Wheat, Geraldine Kauffman, and Harl R. Douglass. Evanston: Row, Peterson and Company, 1951. With Workbook, paper. 96 pp., Book One, $0.72; Workbook, $0.40.

Arithmetic Book One and Workbook One have attractive covers, and they follow the same plan of beautifully colored action pictures as the Primer and Primer Workbook.

The inside covers of Book One have attractive pictures showing "in and out," "between," "middle," etc. The variety of colored objects forming group discussions throughout the book are quite conducive to holding the attention of the child.

Book One seems fine for the average and above average child, but there could be some

difficulty for the below average child because the latter part of the book has more reading, giving instructions to be followed and questions to be answered.

However, the Workbook One is very good and would probably counteract any difficulty the slow child might find in Arithmetic Book One.—MRS. LORENA HOLDER, John H. Reagan School, Dallas, Texas.

Manual, Row-Peterson Arithmetic, Primer and Book One, Margaret Leckie Wheat and Harry G. Wheat. Evanston: Row, Peterson and Company, 1951. Paper, 144 pp., $0.60.

The first sentence in the book, "This Manual is a handbook for teachers," leaves no doubt as to its use and purpose.

Part I. The Arithmetic We Teach, is an over-all picture of what the teacher should know and how to manipulate this knowledge. Chapter 2 discusses "Arithmetic and What It Requires"; Chapter 3 explains "Teaching Number Thinking"; and Chapter 4 treats, "Guidance by the Textbook." These able discussions are applicable to any grade level.

Part II and Part III are devoted to suggestions for using Row-Peterson Arithmetic Primer and Arithmetic Book One, respectively. —Mrs. LORENA HOLDER, John H. Reagan School, Dallas, Texas.

Plane Geometry (Rev. ed.), Frank M. Morgan and W. E. Breckenridge. Boston, Houghton Mifflin Co., 1951. viii+520 pp., $2.32.

Plane Geometry is the 1951 Edition. It was first presented in 1931, and has been revised in 1939 and 1943.

The student has been foremost in the authors' minds in writing this book. The book opens with a Note to the Pupil. In this note, two important ideas are presented: (1) Why study geometry? and (2) The history of geometry. Seventy-five pages are devoted to intuitive geometry thus giving the pupil an opportunity to become familiar with geometric terms.

The outstanding features of the book are: (1) An abundance of exercises after each theorem; (2) Completion tests—True-False tests at intervals throughout the book; (3) Complete explanation of a new topic; (4) Method rules marked so that students can refer back to them; (5) Pictures from industry where geometry is used; (6) Construction problems boxed off so they are easily found; (7) Many interesting construction exercises; (8) Good discussions of locus problems; and (9) One hundred and sixty-one miscellaneous exercises at the close of the book for review.

The reviewer finds this to be one of the most interesting and helpful geometry texts ever reviewed. The abundance of exercises gives variety of material for any class.—CECIL CRUM, Rossville High School, Rossville, Indiana.

Mathematical Snapshots, H. Steinhaus. New York, Oxford University Press, 1950. vi+266 pp., $4.50.

This is a second edition of a book by the same title and the same author, published in 1939. It is a completely revised and enlarged edition.

This is a book containing a wide variety of collections of "curiosa" which would delight any mathematician. However, one must be warned that if he is a "mathematician" not of the "field officer" category (major or any other higher rank) he will not have an easy sailing. This is food for the generals.

On the other hand if one cares to spend a little time and effort in unraveling that which the author "takes for granted that the reader will understand," then he will be rewarded far above his humble expectations. This reviewer was particularly interested in ascertaining whether this book could be recommended to a classroom teacher as possible source material for enrichment purposes. He is delighted to report that the answer to this question is affirmative.

Unfortunately, there is not a table of contents. Still more unfortunate is the omission of an index. Thus, a reader will find some (unnecessary) difficulty when trying to locate some specific examples and illustrations which might be useful for some special classroom purposes. However, this should be construed as a minor criticism.

The topics treated in this book range from the simplest cases such as triangles, squares, games, rectangles, numbers, and tunes to solids (Platonic and Archimedean), geodesics, topological situations, and even the Jordan curve. This reviewer's opinion, and it is his only, differs from the opinion of the author that it is best to avoid (even the simplest) explanation. Nor does he believe that this book can be profitably enjoyed by anyone "with a knowledge of high-school algebra." For example, on page 27 he gives an expression for $\sqrt{2}$ in terms of a continuous fraction with a repeating denominator 2. How many high-school algebra courses treat the topic of continuous fractions?

This book is a masterpiece of printing, but, and this is not a reflection on the author whose "mother tongue" is not English, such is not the case editorially.—A. BAKST, Flushing, New York.

Geometrical Tools, A Mathematical Sketch and Model Book (Rev.), Robert C. Yates. St. Louis, Educational Publishers, Inc., 1949. 194 pages. $3.00.

The general nature of this workbook can perhaps best be described by quoting from the preface. "This book has been designed especially for college students who are prospective teachers of mathematics. It serves not only to focus their attention upon the geometrical tool and the precise manner in which it is used, but also fur-

nishes them with abundant material that can and should be introduced into high school work. The subject matter presented here requires no preliminary knowledge of mathematics in advance of that acquired in the standard freshman courses of algebra, trigonometry, and analytics. . . . The arrangement is based upon the three-hour-per-week class. It is suggested that two of these hours be spent in the classroom, the third in the laboratory. Thus, at the average rate of two plates per week, the material will be found ample for a year course. . . . There are approximately 80 plates, each faced by explanatory text and each designed as a class-hour unit. . . . There is no attempt to encourage mechanical perfection on the part of the student in the art of drafting. Instead, it is hoped that this will bring about a more thorough and sympathetic understanding of geometrical structure."

Among the topics studied are the Straightedge and Modern Compasses, Dissection of Plane Polygons, Compasses Alone, Paper Folding, Straightedge Alone, Linkages, Straightedge with Immovable Figure, Straightedge and Collapsible Compasses, Parallel Ruler, Angle Ruler, Marked Ruler and other Higher Tools.

Each section has an extensive bibliography, and hints about the constructions are generous. The book, however, would not be self-teaching, but would require careful guidance from an instructor. The prospective teacher will find much material in the early part of the book which he could use later in mathematics clubs and as supplementary material for superior students in high school geometry. (Many present high school teachers might find the book valuable for this material.) Whether the appreciation and advanced background obtained from the deeper study of geometrical tools in the latter part of the course is worth the time it would take in a future teacher's crowded schedule is a question. —Henry Swain, New Trier Township High School, Winnetka, Ill.

Better Than Rating: New Approaches to Appraisal of Teaching Services, Commission on Teacher Evaluation of the ASCD—G. S. Willey, Chairman. Washington, D. C., Association for Supervision and Curriculum Development, NEA, 1950. Paper, 83 pp. $1.25.

This thought-provoking bulletin was produced after two years of study by a commission on Teacher Evaluation of the ASCD. The scope of the study is best surmised from the chapter headings, which are as follows: 1) "Teacher Rating: Problems and Issues," 2) "How Teachers Accomplish Best Results," 3) "Evaluation: One Aspect of Professional Growth," 4) "Analysis of Current Teacher-Rating Practices," 5) "How Rating Affects the Schools Program," 6) "A Better Way Than Rating." Within each chapter the authors very clearly outline and discuss the pertinent points. A bibliography is included.

The main thesis is that under present rating procedures, the teacher is on the defensive and usually attempts to conform to practices supposedly desired by the raters. The authors propose a maximum use of democratic practices involved the entire school community—pupils, school people, and lay citizens. Each community is urged to study its own situation and evolve an evaluation plan that will meet the local needs. No ready-made pattern will fit every school community and every program should be continuous and comprehensive. Emphasis is placed on more care in selecting prospective teachers, better in-service guidance and training and guidance out of the profession for those not well fitted for the job. This analysis is a recommended study for anyone who is concerned with personnel evaluation.—Francis R. Brown, Illinois State Normal University, Normal, Illinois.

Science Is a Sacred Cow, Anthony Standen. New York, E. P. Dutton and Co., 1950. 221 pp., $2.75.

The author, a chemist with a background of industrial work and college teaching, administers a severe verbal spanking to the experimental scientists for what he calls their cocksureness and inclination to over-extend the scientific method into areas that are not properly accessible to it. Passing critically from physics through biology, psychology, and the social sciences to mathematics, he rejects the experimental sciences as sources of "absolute truth" and then comes up with the somewhat surprising declaration that "mathematics, as opposed to the rest of science, is really worthwhile and important" and, "in its own limited way, true" because it can be used as a "stepping stone to the really important higher knowledge," which the author interprets in the sense of Platonic philosophy.

The book contains elements of truth, for instance, on the faults of science teaching and the emotional extrapolations of science popularizers, and it is well written. But to this reviewer it appears too superficial, negative, and opinionated to deserve the attention of scientific workers who look for answers to the significant questions: What are the limits of scientific certitude? and Where is the boundary line between knowledge and faith?—Paul R. Neureiter, State Teachers College, Geneseo, New York.

Theory of Probability, M. E. Munroe. New York, McGraw-Hill, 1950. viii+213 pp. $4.50.

Modern probability theory has been inaccessible to the student with only a calculus background because of the central role played by the Lebesgue-Stieljes integral. The presentation of this book is directed at just such a student. He is told that the Lebesgue integral is a generalization of the ordinary integral which can be applied not only to same functions as the ordinary integral (with the same value for the integral) but also to many more functions which are not integrable in the old sense. When a proof of a theorem depends on special prop-

erties of the Lebesgue integral the proof is omitted and only the statement is given. The Lebesgue-Stieljes integral is not used at all.

There remains an account of the theory which in view of the above limitations is surprisingly good. So good, in fact, that the book is to be recommended even to those who are acquainted with the Lebesgue Theory.

The first chapter contains material which is usually presented in courses on college algebra; from this beginning the reader is led gradually to the advanced theory. The last three chapters contain descriptions of the central limit theorem, the Poisson distribution, and the laws of large numbers. The last chapter contains a section on applications to statistical sampling theory.

Among the examples whose solution is given is the famous Buffon needle problem: A board is ruled with equidistant parallel lines and a needle, whose length is less than the distance between the lines, is thrown on the board. What is the probability that the needle touches one of the lines?—M. P. Gaffney, Northwestern University, Evanston, Illinois.

Elements of Mathematical Analysis, Samuel E. Urner and William B. Orange. New York, Ginn and Company, 1950. xi+561 pp., $4.00.

This is another first year of college mathematics text differing from the usual in that it presupposes a knowledge of intermediate algebra and trigonometry (although trigonometry is discussed thoroughly in the text). Hence the student encounters more difficult and advanced material than is the usual case—covering most of the material found in a first year of the calculus.

At Los Angeles City College, where the material of the present volume has been presented, the five-hour freshman course covered all but what is equivalent to the last two chapters—"Formal Integration" and "The Definite Integral and Applications." This was completed in the first semester of the sophomore year.

The authors present the material in a clear, concise manner. There is an unusually large appendix, consisting of reference material from previous work and important advanced topics. There is a lack of historical reference.

Exercises on differentiation are presented early (p. 48) as are those on integration (p. 128). Here, since the technical student will have this information early to apply in other courses, is the outstanding feature of this text.—Joseph Stipanowich, Western Illinois State College, Macomb, Illinois.

What Is Going On?

(*Continued from page 132*)

Mathematics *In* School and *Out*

The Mathematics Club of Crispus Attucks High School of Indianapolis has carried out several projects to stimulate interest in mathematics. The "selling point" was to show the practical application of mathematics to a successful career both *in* school and *out*; so an exhibit and assembly were planned on this basis. Charts which showed the tie-up of mathematics with other courses in high school, then with college courses, and with vocations for life careers were worked out. Measuring instruments, precision tools, and materials which demonstrated mathematical usages and implications used in various classes (every department was represented) were displayed. Pictures of successful graduates who had found mathematics valuable in their chosen field were collected. Included were two engineers, an architect, a draftsman, an air pilot and bombardier, a business executive, an office manager, a laboratory technician, a research chemist, a machine operator, a tool designer, a personnel manager and former member of the War Manpower Commission, a pharmicist, and a mechanic. This aroused a great deal of interest. A program in the auditorium included adult guest speakers who were in business management, personnel service, vocational guidance in social welfare, and government employment service. These men emphasized the importance of mathematical skills in general preparation for jobs in industry as well as the college program and stressed the citizenship value of mathematics. Their contribution was most effective. The Mathematics Club has further projects along this line of guidance.

The last two articles are reported from *The Indiana Mathematics Teacher*, February 1951, Alberta Bogan, Bloomfield, Ind., Editor.

This issue sponsored by the Indianapolis Mathematics Club, Helen R. Pearson, President.

REPORTS FROM THE AFFILIATED GROUPS

JOHN R. MAYOR, *Chairman*
Committee on Affiliated Groups
University of Wisconsin, Madison, Wisconsin

DES MOINES PLANS FOR AFFILIATED GROUPS

Attention of officers of Affiliated Groups is called to the program of the Thirtieth Annual Meeting of The National Council of Teachers of Mathematics which will be held in Des Moines, Iowa, on April 16–19, 1952. Official Delegates should be certified to the Committee on Affiliated Groups no later than February 29.

The two sessions of the Third Delegate Assembly will be held on Thursday, April 17, 8:00–10:00 A.M. and on Friday, April 18, 1:30–3:15 P.M. A luncheon for official Delegates and participants in the program sponsored by the Committee on Affiliated Groups will be held at noon on Friday. Reservations for the luncheon can be made at the time of certification of the Delegates.

The one section of the Des Moines meeting sponsored by the Affiliated Groups will be held from 10:00–11:30 A.M. on Friday. Miss Mary C. Rogers of Westfield, New Jersey, will serve as chairman of the program on the topic:
Specific Programs for the Promotion of Interest in Mathematics Education

1. *In the Community*
2. *Among Students*
3. *Among Teachers of Mathematics.*

Those who will take part on this program will be representatives of Affiliated Groups and schools which have received recognition for activities in these areas.

TRAVELING EXHIBIT

At both the First and the Second Delegate Assemblies, the Delegates indicated their belief that a traveling exhibit of multi-sensory aids sponsored by the Committee on Affiliated Groups would be a useful service to the Affiliated Groups. Miss Madeline Messner, Abraham Clark High School, Roselle, New Jersey, has been appointed chairman of a committee to prepare such an exhibit.

The exhibit will consist of both commercial and teacher-student made devices. Since funds to support the exhibit are very limited, teachers are being asked to donate aids which they have found most useful. Any commercial aids in the exhibit will be donated by the manufacturers.

Those who wish to use the traveling exhibit will pay the cost of transportation to their schools.

If you are willing to contribute to the exhibit or if you would like to plan for the use of the exhibit at some future date, please write to Miss Messner.

Your suggestions will also be welcome.

SPECIAL RECOGNITION FOR TWO AFFILIATED GROUPS

According to action of the First Delegate Assembly, annual renewal of affiliation dues are to be waived for Affiliated Groups with seventy-five per cent or more of their members also members of The National Council of Teachers of Mathematics. Of the fifty Affiliated Groups for the school year 1950–51 only two enjoyed this privilege of having their renewal dues waived. These Groups were the Louisiana-Mississippi Branch of The National Council of Teachers of Mathematics, for which it was reported that one hundred per cent of the members were also members of the national organization, and the Women's Mathematics Club of Chicago and Vicinity for which it was reported that 85.1% of the members were also members of the National Council.

These Groups are also two of the Groups longest affiliated with The National Council. The Louisiana-Mississippi Branch was first affiliated on June 7, 1929, and has Charter No. 3. The Chicago area Women's group was affiliated later that same year on December 28, and was given Charter No. 7.

Two Groups are Added to the Affiliated List

Early in November the Committee on Affiliated Groups approved the applications for affiliation with the National Council of Teachers of Mathematics of the Houston Council of Teachers of Mathematics and the Southern New England Mathematics Association. With these two new Groups the total number of Affiliated Groups was fifty-three on November 10, 1951. By the time this report is published the total will probably be nearly sixty.

The Houston Council is a new organization of teachers from the elementary, junior and senior high school, and college levels in Houston, Texas and vicinity. At the time of application for affiliation nearly seventy per cent of the seventy-five members were also members of The National Council. The Houston Council plans four meetings for this year. Correspondence for the Group should be addressed to Mrs. Mira May Sanders, 1216 Hawthorne St., Houston 6, Texas. The president of the Group is Miss Lel H. Red, Lamar High School, Houston.

The Southern New England Mathematics Association was organized in 1946. The eighty members of the Association live in Connecticut and Western Massachusetts, and are associated with independent boys preparatory schools. Teachers of mathematics from arithmetic through the calculus are represented in the membership. Correspondence for the Group should be addressed to Mr. Albert Nagy, Pomfret School, Pomfret, Connecticut, who is the president of the Association.

Joint Meetings with Local Sections of The Mathematical Association of America

A number of Affiliated Groups have reported successful joint meetings with the sections of The Mathematical Association of America in their areas, and several inquiries have also come from other parts of the country about types of programs that the Affiliated Groups of the National Council and the sections of the Mathematical Association might profitably plan under a joint sponsorship.

The National Council and the Association have long enjoyed close relationships and periodic co-sponsorship of professional studies of great value to both organizations and to all mathematics teachers. The best known activity of this kind is the work done by the Joint Commission of the two organizations. The Report of the Commission was published as the Fifteenth Yearbook of the National Council under the title "The Place of Mathematics in Secondary Education." At present the two organizations have several parallel committees which are giving both separate and joint consideration to common problems, and one Joint Committee which is making plans for a Symposium on Teacher Education in Mathematics.

When there is cooperation on a local basis of groups associated with both of these national organizations, cooperative effort on the national scale will not only be more possible but also more valuable when it takes place. The Committee on Affiliated Groups is anxious to do anything it can to encourage jointly sponsored meetings between the Affiliated Groups and the various sections of the Association. Meetings of this type which have recently come to our attention are listed in the next paragraph. Interested teachers might write to officers of these Groups, as listed in the December, 1951 number of The Mathematics Teacher, for further information.

(*Continued on page 144*)

Candidates for N.C.T.M. Offices—1952 Ballot

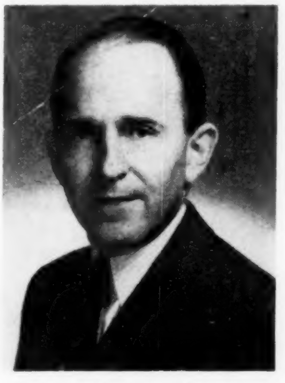
The 1952 Nominating Committee presents the persons listed below as candidates for the designated offices on the Board of Directors. Two candidates are presented for each office. The term of office for Director is three years, and for President and Vice President is two years. However, the office of Vice President representing the field of Junior High School Mathematics is a newly created office, for which election will normally be held during odd-numbered years. This office is therefore to be filled this year for a term of one year.

The Nominating Committee wishes to thank the members of the Council who made suggestions to the Committee. Since not all of the names submitted could be used on the present ballot, it is suggested that names omitted may properly be suggested to the Nominating Committee for 1953.

Lenore John, *Chairman*
E. H. C. Hildebrandt
Houston T. Karnes
Ona Kraft
Veryl Schult

President

Mayor, John Roberts, Professor of Mathematics and Education and Chairman of the Department of Education, University of Wisconsin; also Teacher of Mathematics and Head of Department, Wisconsin High School. S.B., Knox College; A.M., University of Illinois; Ph.D., University of Wisconsin. Instructor in Mathematics, University of Wisconsin in Madison and Milwaukee, 1929–35; Professor of Mathematics and Chairman of Department, Southern Illinois (Normal) University, 1935–47. Member of N.C.T.M.; C.A.S.M.T. (Board of Directors, 1950–; Chairman of Elementary Mathematics Section, 1949–50); Math. Assn. of Am. (Chairman of Ill. Sec., 1939–40; Chairman of Wis. Sec., 1949–50); Am. Math. Soc.; N.E.A.; Wisconsin Mathematics Council; A.A.U.P.; Phi Beta Kappa; Sigma Xi; Pi Mu Epsilon; A.A.A.S. Cooperative Committee on Teaching Science and Mathematics (Representative of M.A.A.); Joint Committee on Symposium in Mathematics Education (Chairman, Representative of M.A.A.); Wisconsin Statewide Curriculum Committee in Mathematics. Director, Conference on Teaching Mathematics, Grades 1–12, University of Wisconsin, 1948–51. Listed in *American Men of Science*, *Who's Who in American Education*. Activities in N.C.T.M. include Member of Board of Directors, 1948–51; Associate Editor of The Mathematics Teacher and Co-editor of the department, "What Is Going On in Your School?"; Chairman of Committee on Affiliated Groups; Member of Committees on Budget, Contests and Scholarships, Place of Meetings, Publicity, and Program (Pittsburgh, Northfield, Stillwater, Gainesville meetings); Chairman of Program and Local Arrangements Committees, Tenth Summer Meeting, Madison, 1950; President of Wisconsin Mathematics Council, 1950–51; Wisconsin State Representative, 1948–51. Publications include "A Generalization of



John Roberts Mayor

the Steiner and Veronese Surfaces," *American Journal of Mathematics*, LVI, 372–80, June, 1934; "Some Suggestions on Teaching Fundamental Mathematical Concepts," *American Journal of Pharmaceutical Education*, XIV, 35–52, January, 1950; Book Reviews in *American Mathematical Monthly*, *National Mathematics Magazine*, *Journal of Educational Research*; and numerous articles in *School Science and Mathematics* and The Mathematics Teacher.

Syer, Henry W., Associate Professor of Education, Boston University, Boston, Massachusetts. S.B., A.M., and Ed.D., Harvard University. Instructor in Mathematics and Science, Gunnery School, Washington, Connecticut, 1937–40; Instructor in Mathematics, Culver Military Academy, Culver, Indiana, 1940–42; U. S. Army, Antiaircraft Artillery (Computor Section) and Signal Corps (Photographic Section), 1942–46; Assistant Professor and Associate Professor of Education, School of Education, Boston University, 1946–; Visiting Professor, School of Education, University of Michigan, Summer, 1951. Member of N.C.T.M.; Association of Teachers of Mathematics in New England (Member of Board of Directors); N.E.A.;

A.A.U.P.; A.A.A.S.; Math. Assn. of Am.; Am. Math. Soc.; Phi Beta Kappa; Phi Delta Kappa. Organizer and first general Chairman, New England Institute for Teachers of Mathematics, 1949. Listed in *Leaders in Education*, *Who's Who in Education*, *Who's Who in the East*. Has served as Curriculum Consultant in Mathematics and speaker at local meetings. Activities in the N.C.T.M. include Member of the Board of Directors, 1948–51; Associate Editor of THE MATHEMATICS TEACHER and Co-editor of the department, "Aids to Teaching"; Chairman of the Committee on Publications of Current Interest; Member of the Committees on Publications and on Evaluation of Films and Filmstrips; Program Chairman for Twelfth Summer Meeting to be held jointly with the New England Institute for Teachers of Mathematics, 1952; participant in programs for conventions. Publications include: "A Classification of Mathematical Instruments and Sources of Their Pictures" and "The Making and Use of

HENRY W. SYER

Motion Pictures for the Teaching of Mathematics" in the *Eighteenth Yearbook* of the N.C.T.M.; co-author of "Thinking," Culver Military Academy, Culver, Indiana; "Mathematical Instruments," (Five booklets), Society for Visual Education; "History of Measure Series" a series of six filmstrips, Young America Films; and articles in *School Science and Mathematics*, THE MATHEMATICS TEACHER, *Film News*, *New Jersey Mathematics Teacher*, and *Audio-Visual Guide*.

Vice President—Elementary School

SAUBLE, IRENE, Director of Exact Sciences, Detroit Public Schools, Detroit, Michigan. A.B., University of Michigan; A.M., University of California. Teacher of Mathematics, Northwestern High School, Detroit, Michigan, 1921–24; Supervisor of Mathematics, Grades 2–12, Detroit Public Schools, 1924–43; Director of Exact Sciences, 1943–. Part-time Associate Professor of Teaching of Mathematics, Wayne University, Detroit, 1936–51; teaching in summer sessions: at Claremont College, Claremont, California, 1931; at Loyola University, Chicago, 1932; at Chico State College, Chico, California,

IRENE SAUBLE

1935; on staff of Workshop at Oakland, California, 1950–51. Member of N.C.T.M.; Phi Beta Kappa; Detroit Mathematics Club; Advisory Committee for Michigan Council of Teachers of Mathematics, 1950. Publications include "The Enrichment of the Arithmetic Course—Utilizing Supplementary Materials and Devices" in the *Sixteenth Yearbook* of the N.C.T.M.; co-author, "The Measurement of Understanding of Elementary School Mathematics," *Forty-Fifth Yearbook, Part I*, National Society for the Study of Education; "Teaching Fractions, Decimals and Per Cent: Practical Applications," *Arithmetic—1947*, University of Chicago Monograph, Number 63; "Applications of the Film in Mathematics," *The Film and Education*, Philosophical Association of America, New York; and articles in *The Instructor*.

BEN A. SUELTZ

SUELTZ, BEN A., State University Teachers College, Cortland, New York. S.B., State College, Aberdeen, South Dakota; A.M. and Ph.D., Columbia University. Teacher of Mathematics and Principal of High School, Trail City, South Dakota; Supervisor of Arithmetic

and Teacher of Mathematics, Cortland Normal School; Professor and Head of Department, Chairman of Graduate Division, State University Teachers College, Cortland, New York. Member of various professional and honor societies; National Survey of Education of Teachers; American Committee of International Commission on Teaching Mathematics; New York State Course of Study Committee (Chairman). Consultant, principally in matters relating to curricula and evaluation in arithmetic. Author of *New Trend Arithmetics; Functional Evaluation in Mathematics Tests; Mathematics for Boys and Girls* (New York State Syllabus); *Mathematics Progress Tests* (New York State); and numerous articles, particularly in evaluation, method, and curriculum in arithmetic. Hobbies: bridge, flowers, American antiques.

Vice President—Junior High School

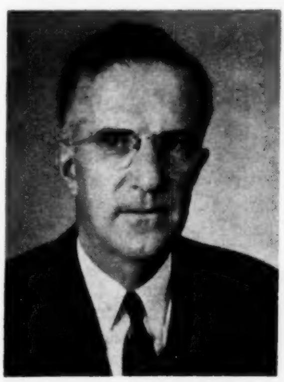
Agnes Herbert

Herbert, Agnes, Clifton Park Junior High School, Baltimore, Maryland. Graduate of Baltimore City Teachers Training College, now Towson State Teachers College, Towson, Maryland; graduate of Teachers College, Columbia University; additional study at Johns Hopkins University; University of Maryland; New York University. Teacher in elementary school, two years; teacher of junior high school mathematics and Chairman of the Department of Mathematics, Clifton Park Junior High School; demonstration teacher at Johns Hopkins University Summer School, 1947; participated in the United States Armed Forces Institute program at Fort George G. Meade, 1943–46. Member of N.C.T.M.; Overseas Teacher Relief Committee for Maryland State Teachers, 1948–49 (Chairman, 1949–50); N.E.A. (member of Resolutions Committee for Maryland 1950–51 and 1951–52); Mathematics Section of the Maryland State Teachers Association (one of organizers, Treasurer for six years, Chairman for two years). Activities in the N.C.T.M. include member of Board of Directors, 1949–52; state representative; Chairman for Twenty-seventh Annual Meeting of N.C.T.M. in Baltimore in 1949; appearance on its convention programs.

Helen A. Schneider

Schneider, Helen A., Oak School, La-Grange, Illinois. Graduate of Chicago Teachers College; additional study (two years), University of Chicago; attended summer workshops, University of Wisconsin and University of Chicago. Teacher in elementary schools in Illinois; teacher of mathematics in Junior High School, La Grange, Illinois; Chairman of Department of Mathematics, District 102, Illinois. Member of N.C.T.M.; Illinois Council of Teachers of Mathematics (President, 1949–50); Committee on Mathematics Sequence (Grades Seven Through Nine) for Schools of Lyons Township, Illinois (Chairman). Has appeared on programs of N.C.T.M. Author of "The Place of Workbooks in the Teaching of Arithmetic," *Arithmetic—1948*, University of Chicago Supplementary Educational Monograph No. 66.

Vice President—Senior High School



Jackson B. Adkins

Adkins, Jackson B., The Phillips Exeter Academy, Exeter, New Hampshire. Ph.B., University of Chicago; Ed.M., Harvard University.

Teacher of Mathematics, Central Junior High School, Lima, Ohio, 1923–25; Bloom Township High School, Chicago Heights, Illinois, 1926–27; Proviso Township High School, Maywood, Illinois, 1927–29; Culver Military Academy, Culver, Indiana, 1929–32; Moses Brown School, Providence, Rhode Island, 1933–39; Phillips Exeter Academy, Exeter, New Hampshire, 1939–. Teacher, U. S. Navy Midshipman School, New York, 1942–44; Head of Mathematics Department, U. S. Navy Pre-midshipman School, Asbury Park and Princeton, New Jersey, 1944–45. Member of N.C.T.M.; Am. Math. Assn.; Association of Teachers of Mathematics in New England (Board of Directors, 1947–50, Speakers Bureau, 1951–52); Southeastern New Hampshire Association of Teachers of Mathematics (Founder and President, 1946–); New Hampshire State Committee on the Revision of the Senior High School Curriculum in Mathematics; Phi Delta Kappa; Tau Kappa Epsilon. Program Chairman, New England Institute for Teachers of Mathematics, 1951; General Chairman, 1952.

WILCOX, MARIE S., George Washington High School, Indianapolis, Indiana. A.B. and A.M. in Mathematics, Indiana University. Teacher of Mathematics, Brown Township High School, Indiana; New Albany High School,

MARIE S. WILCOX

New Albany, Indiana; George Washington High School, Indianapolis, Indiana; Butler University Summer Session, 1945. Member of N.C.T.M.; C.A.S.M.T. (Board of Directors, 1935–40; President, 1939); Mathematics Section, Indiana State Teachers Association (Vice President, 1946; President, 1947); Indiana University Womens Club (President, 1937–38); Indianapolis Panhellenic Association (President, 1930); Phi Mu Fraternity (District President, 1940–43, National Scholarship Director, 1943–48, National Alumnae Vice President, 1948–50); Phi Beta Kappa; Mortar Board; Pi Lambda Theta. Listed in *Leaders in Education;* biography requested for *Who's Who in American Education,* 1951–52 edition. Activities in the N.C.T.M. include Member of Board of Directors, 1948–51; Chairman of Committee on Revision of By-Laws, 1950–51.

Additional Members of the Board of Directors

(Three to be Elected)

ARCHER, ALLENE B., Thomas Jefferson High School, Richmond, Virginia. A.B., Randolph-Macon College; Ed.M., University of Virginia; summer sessions at Harvard University, Columbia University, and Johns Hopkins University. Teacher of Mathematics,

ALLENE B. ARCHER

Hillsboro High School, Tampa, Florida, 1925–33; Richmond Public Schools, 1933–; Head of Mathematics Department, Thomas Jefferson High School, 1951–. Laboratory group leader, Duke Mathematics Institute, 1948–51; New England Mathematics Institute, 1951; Mathematics Laboratory Director, University of Michigan Summer School, 1949–50. Member of N.C.T.M. (Member of Committee on Coordination of Mathematics with Industry, Business, Science and Engineering); Mathematics Section, Virginia Education Association (President, 1949–50); Richmond Curriculum Committee, 1947; Virginia Curriculum Committee, 1950; N.E.A.; Richmond Branch, N.C.T.M. (President, 1951–); Delta Kappa Gamma; A.A.U.W. Co-author of *Plane Geometry Experiments;* author of "Teaching Plane Geometry to Mentally, Physically, and Emotionally Handicapped Pupils," THE MATHEMATICS TEACHER, March, 1951.

BERNHARD, IDA MAY, San Marcos High School (Laboratory School, Southwest Texas State Teachers College), San Marcos, Texas. A.B., M.A., University of Texas; summer sessions at University of Vermont and Columbia University. Teacher in Texas Public Schools, 1927–45; Supervisor of Mathematics in S.W.T.S.T.C. Laboratory School and San Marcos High School, 1945–. Attended Duke University Mathematics Institute, 1946–51. Study Group Leader, Duke University Mathe-

matics Institute, 1950–51; Louisiana State University Mathematics Institute, 1950–51; U.C.L.A. Mathematics Institute, 1951. Member of N.C.T.M. (Member of Committee on Films and Filmstrips); N.E.A.; Math. Assn. of Am.; Texas State Teachers Association (Vice President, Alamo District, 1951–52); A.A.U.W.; Delta Kappa Gamma; Texas Classroom Teachers Association; Texas State Textbook Committee, 1951. Has appeared on programs of N.C.T.M.

IDA MAY BERNHARD

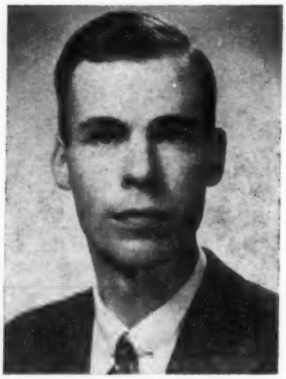
FAWCETT, HAROLD P., Ohio State University, Columbus, Ohio. A.B., Mt. Allison University, Sackville, New Brunswick; M.A. and Ph.D., Columbia University. Teacher of Mathematics, Junior and Senior High School, Ft. Fairfield, Maine, 1914–15; University School, Ohio State University, 1932–47; Home Study Division Y.M.C.A. Schools, New York, 1919–24; Instructor at Columbia University, 1924–32; assistant professor, Ohio State University, 1932–37; associate professor, Ohio State University, 1937–43; professor, Ohio State University, 1943–; Associate director University School, Ohio State University, 1938–41; chairman, Department of Education, 1948–; visiting professor, Northwestern University, summer sessions 1935–40. Member of N.C.T.M.; C.A.S.M.T.; Ohio Mathematics Council; Ohio Education Association; N.E.A.; A.A.U.P.; American Educational Research Association; National Society for the Study of Education; The John Dewey Society; Canadian Universities Association; The Torch Club. Member of Ohio State Mathematics Committee, 1934–35; Committee on Secondary School Curriculum, 1936–38; Committee on Experimental Units, 1942–; Commission on Research and Service of North Central Association, 1943–. Publications include *The Nature of Proof, Thirteenth Yearbook* of the N.C.T.M.; *Mathematics in General Education* (Committee member); and contributions to such journals as THE MATHEMATICS TEACHER, *School Science and Mathematics, Ohio Schools, Educational Research Bulletin, North Central Association Quarterly, The English Journal, California Journal of Secondary Education*, and *The School Executive.*

HAROLD P. FAWCETT

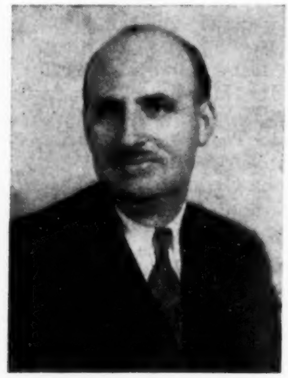
HOEL, LESTA, Supervisor of Mathematics in Grades One to Twelve, Portland Public Schools, Portland, Oregon. Graduate of Washington State Normal School, Cheney, Washington; S.B., Whitman College; A.M., University of Oregon; summer sessions at University of Washington, University of California, University of Colorado. Teacher in rural and elementary schools; teacher of mathematics and head of Department of Mathematics, Grant High School, Portland, Oregon, 1928–40; supervisor of Mathematics, Portland, 1940–; on staff of General Extension Livision of Oregon State System of Higher Education. Member of N.C.T.M.; A.S.C.D.; Oregon Education Association; N.E.A.; Institute of General Semantics

LESTA HOEL

Oregon Council of Teachers of Mathematics (former President); Phi Beta Kappa. Activities in N.C.T.M. include Local Chairman for N.C.T.M. meeting in Portland, 1936; appearance on programs of N.C.T.M. Publications include "Good Teachers Cannot Be Bought," *The Nation's Schools*, February, 1946; "What Constitutes Remedial Work in Arithmetic," THE MATHEMATICS TEACHER, January, 1950.



M. Albert Linton



Albert W. Recht

Linton, M. Albert, The William Penn Charter School, Philadelphia, Pennsylvania. S.B., Haverford College, Haverford, Pennsylsylvania; A.M.T., Harvard University; courses in accounting, Boston University. Teacher of high school Mathematics, Friends Academy, Locust Valley, New York, 1939–41; during World War II, conscientious objector, drafted into Civilian Public Service, 1941–45, and assigned to Massachusetts General Hospital, Boston, as biochemical research assistant; teacher of junior and senior high school mathematics, William Penn Charter School, 1946–; appointed Donald E. MacCormick Master in Mathematics, 1950. Member of N.C.T.M.; Pennsylvania Council of Teachers of Mathematics; Association of Teachers of Mathematics of Philadelphia (President and Official Delegate to Second Delegate Assembly of N.C.T.M.); Harvard Teachers Association. Attended New England Institute for Teachers of Mathematics, 1949.

Recht, Albert W., University of Denver, Colorado. A.B., A.M., University of Denver; Ph.D. in Mathematical Astronomy, University of Chicago. Teacher of Mathematics, Teacher Training, Spanish, Yuma County High School, Colorado, 1920–23; instructor in Mathematics, University of Denver, 1923–32; Director of Chamberlin Observatory, University of Denver, 1926–; professor of Mathematics and Astronomy, University of Denver, 1932–; chairman of Department of Mathematics and Astronomy. Member of N.C.T.M.; A.A.U.P.; American Astronomical Society; N.E.A.; Colorado Education Association (President of Mathematics Section, Eastern Branch, 1948–50); Math. Assn. of Am.; Sigma Xi. Activities in N.C.T.M. include Chairman for Summer Meeting of N.C.T.M., Denver, 1949; delegate from Colorado to annual meetings of N.C.T.M. at Indianapolis, Baltimore, and Chicago; appearance on programs of N.C.T.M. meetings. Publications include several papers in *Popular Astronomy* and *The Astronomical Journal*, including computation of motion of periodic Comet d'Arrest from 1851–1950; "Is Mathematics Out of This World," The Mathematics Teacher, February, 1948.

Affiliated Groups

(*Continued from page 138*)

The Rocky Mountain Section of The Mathematical Association of America meets annually in the spring at the same time and place as the Colorado Council of Teachers of Mathematics and a joint luncheon is held on one of the meeting days. A similar plan is followed in Minnesota by the state groups affiliated with the Council and the Association. In Minnesota one afternoon program is also jointly sponsored, and the two groups choose the luncheon speaker in alternate years.

In Wisconsin a day's program is held each spring for the two state groups. The morning program consists of mathematical topics and the afternoon program of topics on teaching problems. The Kansas groups affiliated with the National Council and the Mathematical Association have found it advantageous to hold a joint morning session and separate afternoon sessions. The Louisiana-Mississippi Branch of the N.C.T.M. has also enjoyed close association with the section of the Association in their area.

Program
The National Council of Teachers of Mathematics

Thirtieth Annual Meeting

Hotel Fort Des Moines, Des Moines, Iowa
April 16, 17, 18, 19, 1952

Host Organization
Iowa Association of Mathematics Teachers

Convention Theme
Improving the Teaching of Mathematics

WEDNESDAY, APRIL 16

9:00 A.M.–12:00 NOON. **Meeting of Board of Directors**—Flamingo Room

2:00 P.M.–5:00 P.M. **Meeting of Board of Directors**—Flamingo Room

7:00 P.M.–9:00 P.M. **Registration**—Lobby

7:30 P.M.–10:30 P.M. **Meeting of Board of Directors**—Flamingo Room

THURSDAY, APRIL 17

8:00 A.M.–9:00 P.M. **Registration**—Lobby

MORNING. **Visiting Des Moines Schools.** (See announcement at end of program.)

8:00 A.M.–10:00 A.M. **Third Annual Delegate Assembly**—Palm Room

Meeting of the official delegates of the affiliated groups.

(The second session is on Friday afternoon.)

Presiding: J. R. MAYOR, The University of Wisconsin, Madison, Wisconsin.

9:00 A.M.–5:00 P.M. **Exhibits**—Lobby and Balcony

9:00 A.M.–10:00 A.M. **Projection of Latest Films**—South Ball Room

10:00 A.M.–11:00 A.M. **Meeting of State Representatives**—Palm Room

Presiding: M. H. AHRENDT, Executive Secretary of The National Council of Teachers of Mathematics, Washington, D. C.

10:00 A.M.–2:00 P.M. **Local Educational Tours** (See announcement at end of program.)

1:30 P.M.–3:30 P.M. **Research Section**—Ranch Room. Sponsored by the Research Committee of the National Council

Presiding: H. VAN ENGEN, Iowa State Teachers College, Cedar Falls, Iowa.

Predicting Success in College Mathematics, FRED ROBERTSON, Iowa State College, Ames, Iowa.

The Use and Evaluation of Textbook Material in the Field of Arithmetic, JEAN F. HAMILTON, Wayne University, Detroit, Michigan.

A Proposed Research Committee Project, HOWARD F. FEHR, Teachers College, Columbia University, New York City.

2:30 P.M.–3:30 P.M. **General Session**—Grand Ball Room

Presiding: HOUSTON T. KARNES, Louisiana State University, Baton Rouge, Louisiana.

A Platform for the National Council, WILLIAM BETZ, Specialist in Mathematics, Rochester, New York.

2:30 P.M.–3:30 P.M. **General Session**—South Ball Room

Presiding: E. H. C. HILDEBRANDT, Northwestern University, Evanston, Illinois.

Relative Values in the Teaching of Mathematics, F. LYNWOOD WREN, George Peabody College for Teachers, Nashville, Tennessee.

2:30 P.M.–3:30 P.M. **General Session**—Palm Room

Presiding: JAMES H. ZANT, Oklahoma

Agricultural and Mechanical College, Stillwater, Oklahoma.

Training Future Mathematicians for America, Virgil S. Mallory, State Teachers College, Montclair, New Jersey.

3:30 p.m.–5:30 p.m. **Arithmetic Laboratory**—Palm Room (See announcement at end of program.)

Leader: Donna Norton, Eugene Field School, Rock Island, Illinois.

3:30 p.m.–5:30 p.m. **Junior High School Laboratory**—Green Room (See announcement at end of program.)

Leader: Emalou Brumfield, Kent State University, Kent, Ohio.

3:30 p.m.–5:30 p.m. **Senior High School Laboratory**—Ranch Room (See announcement at end of program.)

Leader: Henry W. Syer, School of Education, Boston University, Boston, Massachusetts.

4:30 p.m.–5:30 p.m. **Projection of Latest Films**—South Ball Room

7:30 p.m.–9:00 p.m. **General Session of Convention**—Grand Ball Room

Address of Welcome: N. D. McCombs, Superintendent of Schools, Des Moines, Iowa.

Address: *Integers and Equations*, Lyle W. Ashby, Assistant Secretary for Professional Relations, National Education Association, Washington, D. C.

9:15 p.m. **Entertainment, Games, Square Dancing**—Grand Ball Room (See announcement at end of program.)

Friday, April 18

8:00 a.m.–9:00 p.m. **Registration**—Lobby

8:00 a.m.–5:00 p.m. **Exhibits**—Lobby and Balcony

8:30 a.m.–9:30 a.m. **Continuity Section Meetings** (Everyone is urged to attend one of these four continuity sections. See announcement at end of program.)

Elementary Continuity Section—Palm Room

Presiding: Lenore John, Laboratory School, University of Chicago, Chicago, Illinois.

Topic: *Factors Contributing to Difficulty of Learning in Arithmetic.*

Speaker-Analyst: Esther J. Swenson, College of Education, University of Alabama, University, Alabama.

Junior High School Continuity Section—South Ball Room

Presiding: Veryl Schult, Supervisor of Mathematics, Public Schools, Washington, D. C.

Topic: *Caring for Individual Differences in the Junior High School.*

Speaker-Analyst: Rolland R. Smith, Coordinator of Mathematics, Public Schools, Springfield, Massachusetts.

Senior High School Continuity Section—Grand Ball Room

Presiding: Donovan A. Johnson, University of Minnesota High School, Minneapolis, Minnesota.

Topic: *Relational Thinking, Symbolic Thinking, Logical Thinking, on the Senior High School Level.*

Speaker-Analyst: Harold Fawcett, Ohio State University, Columbus, Ohio.

College Continuity Section—Ranch Room

Presiding: William A. Gager, University of Florida, Gainesville, Florida.

Topic: *Good College Teaching—Its Basic Factors*

Speaker-Analyst: Phillip S. Jones, University of Michigan, Ann Arbor, Michigan.

9:45 a.m.–12:00 Noon. **Continuity Discussion Groups** (Registration in one of these groups will indicate your desire to participate in the continuity portion of the program and your intention to attend both the Friday morning and the Saturday morning Continuity Sections which correspond to the discussion group chosen. See announcement at end of program.)

Elem Group—Flamingo Room

Topic: ***Factors Contributing to Difficulty of Learning in Arithmetic.***

Leader: Charlotte W. Junge, College of Education, Wayne University, Detroit, Michigan.

JHS Group A—Room 319

Topic: *Should We Differentiate in Depth and Scope Or in Topics in the Seventh and Eighth Grades? How?*

Leader: Joseph J. Urbancek, Chicago Teachers College, Chicago, Illinois.

JHS Group B—Room 320

Topic: *Who Should Take General Mathematics?*

Leader: R. E. Pingry, University of Illinois, Urbana, Illinois.

JHS Group C—Room 323

Topic: *How Can Ninth Grade Mathematics Be Differentiated From Group to Group To Take Care of Differences in Ability?*

Leader: Mrs. Nanette Blackiston, Supervisor of Junior High School Mathematics, Baltimore, Maryland.

SHS Group A—Room 326

Topic: *Relational Thinking*

Leader: Philip Peak, Indiana University, Bloomington, Indiana.

SHS Group B—Room 335

Topic: *Symbolic Thinking*

Leader: Oscar Schaaf, Ohio State University, Columbus, Ohio.

SHS Group C—Room 337

Topic: *Logical Thinking*

Leader: Kenneth Henderson, University of Illinois, Urbana, Illinois.

Coll Group—Arizona Room

(This is a single group but will break up into three smaller groups after the discussion gets under way.)

Topics and Leaders:

Training the College Teacher, Saunders MacLane, University of Chicago, Chicago, Illinois.

Serving the Students, Bruce Meserve, University of Illinois, Urbana, Illinois.

Designing the Beginning Courses, H. Vernon Price, State University of Iowa, Iowa City, Iowa.

10:00 A. M.–11:45 A.M. **Elementary Section-Panel**—Palm Room

Subject: *Arithmetic-Learning-Psychologist-Mathematician*

Chairman: John R. Clark, Teachers College, Columbia University, New York City.

Other Participants: Leo J. Brueckner, University of Minnesota, Minneapolis, Minnesota; Laura K. Eads, Bureau of Curriculum Research, Public Schools, New York City; Howard F. Fehr, Teachers College, Columbia University, New York City; Maurice L. Hartung, University of Chicago, Chicago, Illinois.

10:00 A.M.–11:45 A.M. **Junior High School Section**—South Ball Room

Presiding: Edith Woolsey, Sanford Junior High School, Minneapolis, Minnesota.

Mathematics and Today's Junior High Schools, Rose Klein, Somers Junior High School, Brooklyn, New York.

Mathematical Objectives for the Crucial Ninth Year, Frank L. Griffin, Reed College, Portland, Oregon.

10:00 A.M.–11:45 A.M. **Learning Aids—Algebra and Geometry Section**—Grand Ball Room

Presiding: Marie S. Wilcox, Washington High School, Indianapolis, Indiana.

Algebra, Learning Aids, and Elementary Scientific Method, Sheldon Myers, Ohio State University, Columbus, Ohio.

Geometry, Learning Aids, and Elementary Scientific Method, John F. Schacht, Bexley High School, Bexley, Columbus, Ohio.

10:00 A.M.–11:45 A.M. **College Section**—Green Room

Presiding: Virgil S. Mallory, State Teachers College, Montclair, New Jersey.

College Mathematics Curriculum Problems, G. Baley Price, University of Kansas, Lawrence, Kansas.

Gains and Failures, and Some Implications, Charles H. Butler, Western Michigan College of Education, Kalamazoo, Michigan.

10:00 A.M.–11:30 A.M. **Affiliated Groups Panel Discussion**—Ranch Room

Presiding: MARY C. ROGERS, Roosevelt Junior High School, Westfield, New Jersey.

Topic: *Specific Programs for the Promotion of Interest In Mathematics Education—In the Community—Among Students—Among Teachers of Mathematics.*

Interest and Talent Development, WALTER H. CARNAHAN, Purdue University, Lafayette, Indiana.

The Mathematics Assembly, BARNETT RICH, Richmond Hill High School, Richmond Hill, New York.

The Mathematics Fair, H. W. CHARLESWORTH, East High School, Denver, Colorado.

Introducing Mathematics to the Faculty, MARY A. POTTER, Supervisor of Mathematics, Racine, Wisconsin.

The Mathematics Institute, An Aid to the Classroom Teacher, HOUSTON T. KARNES, Louisiana State University, Baton Rouge, Louisiana.

The Conference on the Teaching of Mathematics, J. R. MAYOR, The University of Wisconsin, Madison, Wisconsin.

1:30 P.M.–3:15 P.M. **Delegate Assembly—Second Session**—Palm Room

(A continuation of the Thursday morning session of the official delegates of the affiliated groups.)

Presiding: J. R. MAYOR, University of Wisconsin, Madison, Wisconsin.

1:30 P.M.–3:15 P.M. **Foundations of Mathematics**—Grand Ball Room

Presiding: ROBERT E. K. ROURKE, Pickering College, Newmarket, Ontario.

Algebraic Geometry and Geometric Algebra, HENRY W. SYER, Boston University, Boston, Massachusetts.

Using Algebra in Teaching Geometry, HOWARD F. FEHR, Teachers College, Columbia University, New York City.

Using Geometry in Teaching Algebra, BRUCE MESERVE, University of Illinois, Urbana, Illinois.

1:30 P.M.–3:15 P.M. **Junior High School Section**—South Ball Room

Presiding: AGNES HERBERT, Clifton Park Junior High School, Baltimore, Maryland.

Adapting Teaching to Slow Learners, MARY A. POTTER, Supervisor of Mathematics, Public Schools, Racine, Wisconsin.

Ideas Which Have Worked in Stimulating Interest in Certain Topics in Junior High School Mathematics, ETHEL HARRIS GRUBBS, Head of Department of Mathematics, Divisions 10–13, Washington, D. C.

1:30 P.M.–3:15 P.M. **Teacher Education Section**—Ranch Room

Presiding: DALE CARPENTER, Supervisor, Mathematics Education Section, Public Schools, Los Angeles, California.

The Educational System in Finland, VIDAR WOLONTIS, The University of Kansas, Lawrence, Kansas.

The Function of a University Council on Teacher Education in the Preparation of Mathematics Teachers, R. E. PINGRY, University of Illinois, Urbana, Illinois.

1:30 P.M.–3:15 P.M. **Regular Discussion Groups** (Registration in these groups should be made in advance.)

Group No. 1—Room 319

Topic: *Ninth Grade Mathematics—What?—To Whom?—How?*

Leader: EDITH WOOLSEY, Sanford Junior High School, Minneapolis, Minnesota.

Group No. 2—Room 320

Topic: *Recreational Mathematics*

Leader: GUINEVERE D. WHITE, Cordoza High School, Washington, D. C.

Group No. 3—Room 326

Topic: *How Effective Is Our Arithmetic Teaching in Elementary and Junior High School?*

Leader: WARREN F. GEYER, Cole Junior High School, Denver Colorado.

Group No. 4—Room 323

Topic: *How Can Elementary Teachers and Administrators Work Together to Improve Instruction in Arithmetic?*

Leader: GEORGE W. HOHL, Director of Elementary Education, Des Moines, Iowa.

Group No. 5—Room 335

Topic: *Maintaining Children's Interest in Number Concepts.*

Leader: CLARENCE ETHEL HARDGROVE, Northern Illinois State Teachers College, De Kalb, Illinois.

Group No. 6—Room 337

Topic: *The Improvement of Daily Classroom Procedure in Arithmetic.*

Leader: HALE C. REID, Department of Instruction, Public Schools, Cedar Rapids, Iowa.

Group No. 7—Room 324

Topic: *"How Firm a Foundation"?*

Leader: LOIS KNOWLES, University of Missouri, Columbia, Missouri.

3:30 P.M.–5:00 P.M. **College Section—Open Discussion**—Flamingo Room.

Topic: *Can There Be Any Uniformity in the Content of Mathematics for General Education at the College Level?*

Leader: W. L. AYERS, Purdue University, Lafayette, Indiana.

3:30 P.M.–5:30 P.M. **Junior High School Laboratory**—Green Room (See announcement at end of program.)

Laboratory—Junior High School and Plane Geometry.

Leader: FRANCES M. BURNS, Oneida High School, Oneida, New York.

3:30 P.M.–5:30 P.M. **Senior High School Laboratory**—Ranch Room (See announcement at end of program.)

Leader: IDA MAY BERNHARD, Public Schools, San Marcos, Texas.

4:00 P.M.–5:00 P.M. **Projection of Latest Films**—South Ball Room

6:30 P.M.–9:15 P.M. **Banquet**—Grand Ball Room

Address: *Manpower and Mathematics*

Speaker: ALLEN ORTH, Department of Public Relations, General Motors Corporation, Detroit, Michigan.

SATURDAY, APRIL 19

8:00 A.M.–12:00 NOON. **Registration**—Lobby

8:00 A.M.–12:00 NOON. **Exhibits, Commercial and Instructional**

8:30 A.M.–9:15 A.M. **Continuity Sections** (Everyone is urged to attend one of these four continuity sections. See announcement at end of program.)

Elementary Continuity Section—Palm Room

Presiding: LENORE JOHN, Laboratory School, University of Chicago, Chicago, Illinois.

Topic: *Factors Contributing to Difficulty of Learning Arithmetic.*

Speaker-Analyst: ESTHER J. SWENSON, College of Education, University of Alabama, University, Alabama.

Junior High School Continuity Section—South Ball Room

Presiding: VERYL SCHULT, Supervisor of Mathematics, Public Schools, Washington, D. C.

Topic: *Caring for Individual Differences.*

Speaker-Analyst: ROLLAND R. SMITH, Coordinator of Mathematics, Springfield, Massachusetts.

Senior High School Continuity Section—Grand Ball Room

Presiding: DONOVAN A. JOHNSON, University of Minnesota High School, Minneapolis, Minnesota.

Topic: *Relational Thinking, Symbolic Thinking, Logical Thinking, on the Senior High School Level.*

Speaker-Analyst: HAROLD FAWCETT, Ohio State University, Columbus, Ohio.

College Continuity Section—Ranch Room

Presiding: WILLIAM A. GAGER, University of Florida, Gainesville, Florida.

Topic: *Good College Teaching—Its Basic Factors*

Speaker-Analyst: PHILLIP S. JONES,

University of Michigan, Ann Arbor, Michigan.

9:30 A.M.–11:00 A.M. **Elementary Section**—South Ball Room

Presiding: MARY C. ROGERS, Roosevelt Junior High School, Westfield, New Jersey.

What is Meaningful Arithmetic? ROBERT H. KOENKER, Ball State Teachers College, Muncie, Indiana.

What Are the Characteristics of a Program for Meaningful Arithmetic? FOSTER E. GROSSNICKLE, New Jersey State Teachers College, Jersey City, New Jersey.

9:30 A.M.–11:00 A.M. **Class Teaching Demonstration—Senior High School**—Grand Ball Room

Presiding: CARL N. SHUSTER, State Teachers College, Trenton, New Jersey.

Teacher: EMIL J. BERGER, Monroe High School, St. Paul, Minnesota.

Demonstration: *The Teaching of the Second Year of High School Algebra.*

Participants: Students from Monroe High School, St. Paul, Minnesota.

9:30 A.M.–11:00 A.M. **Senior High School Section**—Palm Room

Presiding: ORVILLE A. GEORGE, Public Junior College, Mason City, Iowa.

How Can a Teacher of High School Mathematics Make Practical Use of Source Units? H. C. TRIMBLE, Iowa State Teachers College, Cedar Falls, Iowa.

Teaching Mathematics for Appreciation, DANIEL B. LLOYD, Wilson Teachers College, Washington, D. C.

9:30 A.M.–11:00 A.M. **College Section**—Ranch Room

Presiding: J. R. MAYOR, University of Wisconsin, Madison, Wisconsin.

The Impact of Research on College Algebra and the College Curriculum, SAUNDERS MACLANE, University of Chicago, Chicago, Illinois.

College General Mathematics, KENNETH E. BROWN, University of Tennessee, Knoxville, Tennessee.

9:30 A.M.–11:00 A.M. **History of Mathematics**—Green Room

Presiding: HENRY W. SYER, Boston University, Boston, Massachusetts.

The Development of Methods of Solving Linear Equations, PHILLIP S. JONES, University of Michigan, Ann Arbor, Michigan.

Enriching the Teaching of Linear Equations by Examples of Historical Methods, JAMES H. ZANT, Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma.

9:30 A.M.–11:15 A.M. **Regular Discussion Groups** (Registration for these discussion groups should be made in advance. See announcement at end of program.)

Group No. 8—Room 326

Topic: *How to Teach to Provide for Training in Thinking.*

Leader: K. B. HENDERSON, University of Illinois, Urbana, Illinois.

Group No. 9—Room 335

Topic: *What Practical Provisions Can Be Made in a Public School Curriculum For the Accelerated Child?*

Leader: BURNETT SEVERSON, Morey Junior High School, Denver, Colorado.

Group No. 10—Room 319

Topic: *Reading Difficulties in Mathematics Learning—What To Do About Them.*

Leader: CATHERINE A. V. LYONS, Perry High School, Pittsburgh, Pennsylvania.

Group No. 11—Room 337

Topic: *How Can General Mathematics Above the Eighth Grade Level Be Given the Status of Respectability of the Sequential Courses?*

Leader: HUMPHREY C. JACKSON, Parcells Junior High School, Grosse Pointe, Michigan.

Group No. 12—Room 320

Topic: *How Dynamic Can Geometry Become?*

Leader: RODERICK C. MCLENNAN, Arlington Heights Township High School, Arlington Heights, Illinois.

Group No. 13—Room 323
Topic: *Glamorizing Mathematics*
Leader: NAOMI HICKS, Boone High School, Boone, Iowa.

Group No. 14—Room 324
TOPIC: *How Can We Meet the Challenge, "But It Isn't Practical"?*
Leader: REX E. HARVEY, Elkhart High School, Elkhart, Indiana.

11:00 A.M.–12:00 NOON. **Business Meeting of the NCTM**—South Ball Room

12:30 P.M.–2:00 P.M. **Convention Luncheon**—Grand Ball Room

2:30 P.M.–4:00 P.M. **Class Teaching Demonstration—Elementary**—South Ball Room
Presiding: MARY HANNUM, Park Avenue School, Des Moines, Iowa.
Teacher: IDA MAE HEARD, Southwestern Louisiana Institute, Lafayette, Louisiana.
Demonstration Lesson: *Your Town and Mine.*
Participants: Fifth grade pupils from Park Avenue School, Des Moines, Iowa.

2:30 P.M.–4:00 P.M. **Elementary and Junior High School Section**—Ranch Room (This section is sponsored in part by the Wisconsin Mathematics Council.)
Presiding: DOROTHY SWARD, Roosevelt Junior High School, Beloit, Wisconsin.
Let's Stop Confusing Our Children, RALPH J. COOKE, Director of Elementary Education, Fond du Lac, Wisconsin.
Computation With Approximate Data, Beginning in the Seventh Grade, CARL N. SHUSTER, State Teachers College, Trenton, New Jersey.

2:30 P.M.–4:00 P.M. **Senior High School Section**—Grand Ball Room
Presiding: LUCY E. HALL, Wichita High School North, Wichita, Kansas.
Meanings and/or Mechanics in Algebra, H. C. CHRISTOFFERSON, Miami University, Oxford, Ohio.
The Treatment of Axioms and Postulates When Beginning Geometry, JOSEPH A. NYBERG, Morgan Park High School, Chicago, Illinois.

2:30 P.M.–4:00 P.M. **Secondary Section**—Palm Room
(This section is sponsored by the Nebraska Section, National Council of Teachers of Mathematics.)
Presiding: MAUDE HOLDEN, Nebraska Representative for the NCTM, Ord, Nebraska.
The Relative Merits of Algebra and General Mathematics in the Development of Mathematical Literacy, MILTON W. BECKMANN, Teachers College High School, University of Nebraska, Lincoln, Nebraska.
Consumer Mathematics in the Twelfth Grade, RICHARD R. SHORT, Public Schools, Lincoln, Nebraska.

ANNOUNCEMENTS

Registration

The registration fee is fifty cents for members of The National Council of Teachers of Mathematics, members of the Mathematical Association of America, and for teachers in elementary schools. The fee for non-members and visitors is $1.50. Undergraduate students sponsored by a faculty member, relatives of members, invited speakers who are not members, members of the press, and commercial exhibitors are not charged the registration fee but should register. You are urged to register in advance, but, if you do, please check in at the Registration Desk upon your arrival at the meeting. Use the Advance Registration and Reservation Form. (See page 154.) Registrations and reservations received before April 5 will be acknowledged by return mail.

Hotel Reservations

	single	double
Fort Des Moines	$3.75	$6.00*
Brown	3.50	5.00
Kirkwood	3.50	5.00*
Savery	4.50	6.50*
Randolph	3.75	5.75
Elliott	3.00	4.50

All rates with bath. Rooms without bath, about $1.00 less.

* No rooms without bath.

Information

The Information Committee will furnish you with information on rooms, meals, parking, amusements, stores, schools, colleges, etc. A daily bulletin of special activities will be posted. When at the convention, ask for help at the Information Desk.

Banquet and Luncheon Reservations

Reservations for the banquet on Friday and for the luncheon on Saturday should be made in advance. Requests should be accompanied with check or money order. All orders received before April 5 will be acknowledged by return mail. Banquet, \$4.00; Luncheon, \$2.25, tax and tips included. Use the Advance Registration and Reservation Form.

Continuity Sections and Discussions

On Friday and Saturday mornings, ***four*** Continuity Sections, ***Elementary, Junior High School, Senior High School,*** and ***College,*** have been arranged. (Everyone is invited to attend one of these.)

Immediately following these Continuity Sections on Friday morning, the Continuity Discussion Groups (25 to a group, those who have registered for them) will meet for the discussion of the problem presented by the speaker of the corresponding Continuity Section. On Saturday morning, these Continuity Discussion Groups (one from the Elementary Section, three from the Junior High School Section, three from the Senior High School Section, and one from the College Section) will make reports to their respective Continuity Sections at which time the speaker of the Friday morning Continuity Section will act as analyst. He will evaluate the reports, relate them to the problem under consideration, and make further contributions to the problem. (Everyone is invited to attend one of these Continuity Sections on Saturday.)

If you wish to follow through with this phase of the convention program, you will register for the Continuity Discussion Group of your choice and attend the corresponding Continuity Section meeting on both Friday morning and Saturday morning. For example, if you register for one of the three Senior High School Continuity Discussion Groups, you will plan to attend the Senior High School Continuity Section on Friday morning and again on Saturday morning. Registration in the Continuity Discussion Groups will be limited. Use the Advance Registration and Reservation Form. (See page 154.) Admittance cards will be sent to those making request before April 5.

Regular Discussion Groups

These are scheduled for Friday afternoon and Saturday morning. The number in each group is limited to twenty-five. Registration should be made in advance. Use the Advance Registration and Reservation Form. Admittance cards will be sent to those making reservations before April 5.

Visiting Des Moines Schools

You are cordially invited to observe classes in mathematics to be conducted in both public and parochial schools on Thursday morning, April 17. There will be an opportunity to observe the teaching of arithmetic in elementary and junior high schools, as well as mathematics at the senior high school and college levels. The latter will include classes in practical mathematics, applied mathematics, and socialized arithmetic at the Des Moines Technical High School, Dowling High School, and Drake University. A detailed schedule will be available at the Information Desk, but requests to observe should be mailed in advance to George W. Hohl, 629 Third Street, Des Moines 9, Iowa.

Mathematics Laboratories

Five laboratory periods, each two hours in length, have been scheduled, *three* on Thursday afternoon and ***two*** on Friday afternoon. Each member of the group will be given the opportunity of making one or

more models under the direction of the leader of the group. A minimum cost charge for materials used will be made. The number permitted to register in these groups will be limited. You may register in one or two of these laboratory periods. Register in advance. Use the Advance Registration and Reservation Form. Admittance cards will be mailed to those whose requests are received by April 5.

Educational Tours

Seven tours have been planned for Thursday morning, 10:00 A.M. to 2:00 P.M. Note that one person can take but one tour. Since numbers are limited, second choices may be wise. The right to cancel and return money paid in advance is reserved. The tours are:

No. 1. The Municipal Airport and Weather Station, luncheon in the "Cloud Room." Total cost, $1.75.

No. 2. Drake University, new science buildings, presentation of "opportunities in mathematics and actuarial science," luncheon. Total Cost, $1.00.

No. 3. Hy-Line Poultry Farms, Iowa countryside, scientific farming, "old-fashioned chicken dinner." Total cost, $1.50.

No. 4. Des Moines Art Center, gallery show featuring modern furniture design, choice of luncheon at Younkers. Transportation, 60 cents.

No. 5. Central National Bank, modern banking procedures, special movie. No cost.

No. 6. Equitable Life Insurance Company of Iowa, electronic computer, actuarial procedures, view of city from roof. No cost.

No. 7. Meredith Publishing Company, operations connected with publishing modern magazines and color pictures. No cost.

Visitors will be welcome at the Drake Municipal Observatory on Wednesday night. Arrange your own transportation and come if the sky is clear.

Supplies and Equipment

Speakers and other participants on the program who need blackboards, projection equipment or other materials should communicate with Mr. Clifton Schropp, 629 Third Street, Des Moines, Iowa, before April 1.

Commercial Exhibits

Textbooks and commercial teaching aids will be on the mezzanine at Hotel Fort Des Moines from Thursday morning at 9 o'clock to noon on Saturday. Inquiries for exhibit space should be addressed to Dr. O. C. Kreider, Department of Mathematics, Iowa State College, Ames, Iowa.

School Exhibits

There will be an exhibit of mathematics models, instruments, teaching aids, and other classroom materials on the mezzanine at Hotel Fort Des Moines from Thursday morning at 9 o'clock to Saturday noon. Teachers who wish to exhibit their materials are requested to communicate with Miss Ruth Miller, Ames High School, Ames, Iowa.

Films and Filmstrips

Some of the latest mathematical films and filmstrips will be projected in the South Ballroom on Thursday morning and afternoon and Friday afternoon.

Meals for Special Groups

If any group wishes to arrange for a dinner on Thursday, a breakfast on Friday or Saturday, or a luncheon on Friday, the local committee will help make these arrangements. Write to Dr. H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa.

Location of Meeting Rooms

Hotel Fort Des Moines is headquarters for the convention. All rooms and parlors used for meetings and discussions are in the hotel.

Mail and Telegrams

Mail and telegrams for those attending the convention should be addressed in care of The National Council of Teachers of Mathematics, Hotel Fort Des Moines, Des Moines, Iowa. Mail may be obtained at the Registration Desk.

Refunds on Reservations

No ticket refunds will be made later than three hours prior to the function for which reservation was made, i.e., tours, luncheon, banquet.

Certificate of Attendance

If you care to take back to your school authorities a statement certifying your attendance at the convention, make request for this at the Registration Desk on Saturday morning.

Fun Night

On Thursday night, following the general session of the convention, the Reception Committee has planned an informal evening of fun for all. There will be "ice breakers," games, square dancing, and entertainment. Square dancing demonstrations will be furnished by the Des Moines Recreation Commission.

PROGRAM COMMITTEE

Chairman—H. W. Charlesworth, Denver, Colorado; Emil J. Berger, St. Paul, Minn.; Frances M. Burns, Oneida, N. Y.; Dale Carpenter, Los Angeles, Calif.; John R. Clark, New York City; Harold P. Fawcett, Columbus, Ohio; Kenneth Henderson, Urbana, Ill.; E. H. C. Hildebrandt, Evanston, Ill.; Lenore John, Chicago, Ill.; Donovan A. Johnson, Minneapolis, Minn.; Phillip S. Jones, Ann Arbor, Mich.; Joy E. Mahachek, Indiana, Pa.; Virgil S. Mallory, Montclair, N. J.; J. R. Mayor, Madison, Wis.; Philip Peak, Bloomington, Ind.; G. Baley Price, Lawrence, Kan.; Carl N. Shuster, Trenton, N. J.; Henry W. Syer, Boston, Mass.; H. Van Engen, Cedar Falls, Iowa; Alan Wayne, New York City.

LOCAL COMMITTEES

Central Planning Committee

Chairman—H. Van Engen, Cedar Falls; Ethel Cain, Des Moines; Lamont Constable, Mason City; Mary Jo Cosgrowe, Cedar Rapids; Ruth Davison, Des Moines; O. C. Kreider, Ames; Ruth Miller, Ames; H. Vernon Price, Iowa City; Viola Smith, Waterloo.

Registration Committee

Co-Chairmen—Irvin H. Brune, Cedar Falls, and H. C. Trimble, Cedar Falls; Bernice Bernatz, Fort Dodge; Hilda C. Dethlefs, Sioux City; Gladys Grimes, Charles City; Don Inman, Keokuk; James McFadgen, Emmetsburg; Rose Marty, Sioux City; Ross Nielsen, Hudson; Dorothy Pearson, Sioux City; Mona Redmond, Sioux City; Lois M. Staker, Ruthven; Loretta Van Ness, Sioux City.

Publicity Committee

Chairman—Lamont Constable, Mason City; Bernice Bernatz, Fort Dodge; Howard Elmore, Cedar Falls; Orville A. George, Mason City; Katherine Walker, Mason City.

Local Arrangements Committees

Co-Chairmen—Ethel Cain, Des Moines, and Ruth Davison, Des Moines.

Audio-Visual and Equipment Committee

Chairman—Clifton Schropp, Des Moines; Waldemar Gjerde, Cedar Falls; John Hedges, Iowa City; Harold Kooser, Ames; Ralph Tomlinson, Des Moines.

Educational Tours Committee (all of Des Moines)

Co-Chairmen—Earl Canfield and Violet Sherwood; Claude McBroom; Maurice Lewis; Joseph Sisbeau.

Reception Committee (all of Des Moines)

Chairman—Katheryn Stewart, Ted

Ellgaard, Don Emmanuel, Beulah Newton.

School Visitation Committee (all of Des Moines)

Chairman—George Hohl, B. E. Gillam; Arthur Landry; Sister Mary James Paul, B.V.M.; Sister Mary Eileen Therese, B.V.M.; J. Edgar Stonecipher.

Information Committee (all of Des Moines)

Chairman—Hazel Hope; Martha Bjornson; Mansel Burham; Durward Crisman; Lester Gabel; Maude Heffernan; Clarence Irwin; Geraldine Rendleman; Lydia Rogers; Sister Rita Joseph, O.P.; Sister Beatrice Marie, R. S. M.; Sister Mary Josetta, B.V.M.; Sister Mary Thaddeus, B.V.M.; Kenneth Wittkop.

Hospitality Committee

Chairman—Faye Kinkennon, Des Moines; Ida Braden, Cedar Rapids; W. G. Hatfield, Independence; Dorothy Horn, Des Moines; Mrs. Lucile Johnson, Des Moines; Margaret McEniry, Des Moines; Shirley Nolte, Keokuk; Mary Rickey, Cedar Rapids; Mrs. Dora Telleir, Fort Dodge; Ruth Wedgewood, Sioux City.

Printing—Signs—Teaching Materials

Chairman—Viola Smith, Waterloo; Donald DeJager, Newton; Hester Douthart, Newton; Howard Elmore, Cedar Falls; E. W. Hamilton, Cedar Falls; William Maricle, Cedar Falls; Mrs. Winnie Palmer, Newton; Vera Tussing, Marshalltown.

Banquet Committee

Chairman—H. Vernon Price, Iowa City; Ralph Aschenbrenner, Iowa City; Robert Castater, Iowa City; Marion Comwall, Marshalltown; Ruth Greenwald, Iowa City; Dorothy Horn, Des Moines; Lucille Houston, Burlington; Melvina Swanson, Davenport; Marlyn Wolleson, Independence.

Luncheon Committee

Chairman—Mary Jo Cosgrove, Cedar Rapids; Darlene Coffman, Cedar Rapids; Paul Jones, Cedar Rapids; Melvina Swanson, Davenport.

ADVANCE REGISTRATION AND RESERVATION FORM

Please fill out legibly and mail with remittance to H. C. Trimble, Iowa State Teachers College, Cedar Falls, Iowa, before April 5, 1952. Make check or money order payable to convention treasurer, Irvin H. Brune.

Mr.-Miss-Mrs. ____
Last Name First Initial

Home Address ____
Street and Number City Zone State

School Address ____
Name of School City State

Position ____

Place an X before the address to which you wish your tickets sent.

Registering as: Member of NCTM ________ Member of MAA ________ Elementary Teacher ________ Student ________ Exhibitor ________ Non-Member ________

Check your field of interest: Elementary ________ J.H.S. ________ S.H.S. ________ College ________ Teacher Education ________ Supervision ________ Other ________

(Reverse side, please.)

School Exhibits

Chairman—Ruth Miller, Ames; Russell Drumright, Cedar Falls; Izac Emrick, Des Moines; Inez Gwynn, Shenandoah; H. G. Hazelett, Washington; Margaret Helt, Sioux City; Naomi Hicks, Boone; Kathryn Hinsenbrock, Charles City; Roy Jorgensen, Sac City; Robert Kilgore, Keokuk; Gladys Kluever, Atlantic; Della McMahon, Hudson; Nora Moss, Fort Dodge; Claude Schnell, Des Moines; Edna Schuler, Ames; Carl Wehner, Gladbrook; Alice White, Dubuque.

Commercial Exhibits

Chairman—O. C. Kreider, Ames; Frieda Blum, Fort Dodge; Mary Hannum, Des Moines; Williard Kacena, Des Moines; Herschel Smith, Red Oak.

Our Public Relations

(*Continued from page 101*)

J. W. N. Sullivan, in his essay, "Aspects of Science," calls mathematics "an art, and a great art. It is on this, besides its usefulness in practical life, that its claim to esteem must be based."

Richardson explains, "The creative mathematician uses imagination and intuition to conjecture new results and new methods of research, but he does not assert that his guesses are correct until he has succeeded in proving them logically."

In *Mathematics and the Imagination*, Kasner and Newman give a colorful summary of the attributes of mathematics. Let us share these concepts with our pupils, with the public, Then we will not be asked, "Why should mathematics be included in the curriculum?"

In mathematics we have a universal language, valid, useful, intelligible everywhere in place and in time—in banks and insurance companies, on the parchments of the architects who raised the Temple of Solomon, and on the blueprints of the engineers who, with their calculus of chaos, master the winds. Here is a discipline of a hundred branches, fabulously rich, literally without limit in its sphere of application, laden with honors for an unbroken record of magnificent accomplishments. Here is a creation of the mind, both mystic and pragmatic in appeal. Austere and imperious as logic, it is still sufficiently sensitive and flexible to meet each new need. Yet the vast edifice rests on the simplest and most primitive foundations, is wrought by imagination and logic out of a handful of childish rules.

Registration Fee	$0.50 or $1.50	______	Amount ______
Banquet, Friday	Number @ $4.00	______	Amount ______
Luncheon, Saturday	Number @ $2.25	______	Amount ______
Tour No. ______	Number @ ______	______	Amount ______
			Total Amount $______

Discussion Groups (Regular and Continuity)

First Choice: Group ______________ Leader ______________

Second Choice: Group ______________ Leader ______________

Mathematics Laboratories (Register for one or two.)

Thursday Arithmetic __________ Thursday J.H.S. __________ Thursday S.H.S. __________

Friday J.H.S. __________ Friday S.H.S. __________

Enrollments in Discussion Groups and in Mathematics Laboratories will be made as registrations come in and number enrolled will be limited.

If you need more of these advance registration and reservation forms, or copies of the entire program please make request for them to H. Van Engen, Iowa State Teachers College, Cedar Falls, Iowa.